Math 235, Fall 2001

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Homework Set No. 1, solutions.

1. (1) $1-(5/6)^{10}$. (2) $1-2\cdot(5/6)^{10}+(2/3)^{10}$. (3) $\binom{10}{3}\binom{7}{2}6^{-10}$.

2. $\frac{1}{6}\cdot\frac{1}{\binom{52}{4}}+\frac{1}{6}\cdot\frac{5}{\binom{52}{4}}+\frac{1}{6}\cdot\frac{\binom{6}{4}}{\binom{52}{4}}=\frac{7}{2\binom{52}{4}}$.

3. $P(E_1)=5p^4(1-p)+p^5=5p^4-4p^5$ and $P(E_2)=1-(1-p)^5-5p(1-p)^4-\binom{4}{2}p^2(1-p)^3-p^3(1-p)^2$. As $E_1\subset E_2$, E_1 and E_2 are not independent.

4. Let $E_1=\{\text{no red toy}\}$, $E_2=\{\text{no blue toy}\}$, $E_3=\{\text{no green toy}\}$. Then $P(E_i)=(1-p/3)^5$, $P(E_i\cap E_j)=(1-2p/3)^5$ for $i\neq j$ and $P(E_1\cap E_2\cap E_3)=(1-p)^5$, so $P(E_1^c\cap E_2^c\cap E_3^c)=1-P(E_1\cup E_2\cup E_3)=1-3(1-p/3)^5+3(1-2p/3)^5-(1-p)^5$.

5.

$$P(\text{ticket 1 wins})=P(\text{two from each line})+P(\text{two from one line, three from another})=\frac{3\cdot 3\cdot 44+2\cdot 3}{\binom{50}{5}}=\frac{402}{\binom{50}{5}}$$

$$P(\text{ticket 2 wins})=P(\text{two from first line})+P(\text{three from first line})=\frac{3\cdot\binom{47}{3}+\binom{47}{3}}{\binom{50}{5}}=\frac{49726}{\binom{50}{5}}$$

$$P(\text{ticket 3 wins})=P(\text{2, 3 both chosen})+P(\text{one of 2,3 chosen})=\frac{\binom{48}{3}+2\cdot\binom{46}{2}}{\binom{50}{5}}=\frac{19366}{\binom{50}{5}}$$

$$P(\text{ticket 4 wins})=P(\text{3 chosen})+P(\text{3 not chosen})=\frac{(4\binom{45}{2}+4\cdot 45+1)+45}{\binom{50}{5}}=\frac{4186}{\binom{50}{5}}$$

6. This is the same as choosing an $r\times n$ matrix in which every entry is independently 0 or 1 with probability 1/2 and ending up with at most one 1 in every column. Since columns are independent, this gives $((1+r)2^{-r})^n$

7. Call the teams A and B, and assume your client wants to bet on A. In this solution, *betting on a sequence* of games (say AABAABA) means that we place an initial amount x on A; if A wins, we place the entire amount $2x$ on A (if A loses, we quit); if A wins, we place the entire amount $4x$ on B, etc. Of course the games end after the 5th game in this example, at which point we just stop betting.

Now bet $1000/2^6$ on every one of such sequences in which A wins. Note that there are 2^6 such sequences. Note also that this is equivalent to betting $1000/2^3$ on AAAA, $1000/2^4$ on each of AAABA, AABAA, ABAAA, BAAAA, etc. Finally, note that if one of such sequences wins, it nets you 2^7 times the initial capital.