Homework 5
Math 128A
Due Friday, 12/05/08, 11:00 a.m.

1. (a) For the data below find the least squares polynomials of degrees 1, 2 and 3. You may find the MATLAB commands `polyfit` and `polyval` useful.
   (b) Make plots of the data and the polynomials on the same graph.
   (c) For each least squares polynomial, $P(x)$, compute the error
   $$E_2 = \sum_j \left(y_j - P(x_j)\right)^2 .$$
   (d) Which polynomial do you think best approximates the data? Does increasing the degree of the least squares polynomial always decrease the error? Does increasing the degree of the least squares polynomial give a better approximation? Explain.

x	0.000	0.143	0.286	0.429	0.571	0.714	0.857	1.000
y	0.103	0.121	0.179	0.250	0.444	0.519	0.897	1.269

2. Let $f(x) = (x+1)\exp\left(\dfrac{-3(x+1)^2}{4}\right)$.
   (a) Find the third degree Taylor polynomial, $S(x)$, of $f(x)$ about $x = 0$.
   (b) Find the third degree polynomial, $P(x)$, that minimizes
   $$\int_{-1}^{1} \big(f(x) - P(x)\big)^2 \, dx.$$
   Solve this problem by using orthogonal polynomials. Perform the necessary integrations using numerical quadrature.
   (c) Find the third degree polynomial, $Q(x)$, that minimizes
   $$\int_{-1}^{1} \big(f(x) - Q(x)\big)^2 w(x) \, dx,$$
   where $w(x) = (1-x^2)^{-1/2}$. Solve this problem by using orthogonal polynomials. Perform the necessary integrations using numerical quadrature. The change of variables $x = \cos(\theta)$ (or $\theta = \cos^{-1}(x)$) eliminates the singularity in the integrand.
   (d) Make plots of f, S, P, and Q, and plots of the differences $f - S$, $f - P$, and $f - Q$ on the interval $[-1, 1]$. Comment on the different approximations? What are the advantages of each of the approximations?

The first four Legendre polynomials, $L_j(x)$, and Chebyshev polynomials, $T_j(x)$, are

$$\begin{array}{ll} L_0(x) = 1 & T_0(x) = 1 \\ L_1(x) = x & T_1(x) = x \\ L_2(x) = (3x^2 - 1)/2 & T_2(x) = 2x^2 - 1 \\ L_3(x) = (5x^3 - 3x)/2 & T_3(x) = 4x^3 - 3x \end{array}$$