## Math 207C Homework 4 Due Friday, May 4th

1. A chemical is transported along a pipe of length L by advection with constant positive velocity (a > 0) and by diffusion (D > 0). The concentration is held at a nonzero constant value (A > 0) at the left boundary and zero at the right boundary. The equations for this system are

$$c_t + ac_x = Dc_{xx},$$
  
$$c(0,t) = A, \quad c(L,t) = 0.$$

- (a) Give two different nondimensionalizations based on the two choices of time scale. In each case give the physical meaning of the time scale. Both should involve the dimensionless parameter Pe = aL/D (called Peclet number).
- (b) Which equation is appropriate for the limit  $Pe \to 0$ , and which is appropriate for  $Pe \to \infty$ ? Explain.
- (c) In the limit of large and small *Pe*, where are the boundary layers (if any) in the two problems? What boundary conditions should be satisfied by the leading order outer solution?
- 2. Compute the leading order composite expansion to the problem

$$\epsilon u'' + \sqrt{x}u' - u = 0,$$
  
 $u(0) = 0, \quad u(1) = e^2.$ 

3. In class we constructed the leading order composite expansion to the initial value problem

$$\epsilon \ddot{u} + \dot{u} + u = 0,$$
  
$$u(0) = 0, \quad \epsilon \dot{u}(0) = 1.$$

- (a) Find the terms at order  $\epsilon$  for the inner and outer expansions, perform matching at this order using the intermediate scale, and give the composite expansion.
- (b) Compute the exact solution to this problem. Use it to assess the accuracy of the leading order composite expansion and the expansion from part (a) for different values of  $\epsilon$ .