Math 207C
Homework 8
Due Friday, June 8th, 5:00 PM

1. (a) Use the method of matched asymptotic expansions to construct the leading order composite solution as $\epsilon \to 0$.

$$\begin{aligned} \epsilon u'' + (1+x)u' + u &= 0 \\ u(0) &= 0 \\ u(1) &= 1 \end{aligned}$$

   (b) Use WKB to construct the leading order solution as $\epsilon \to 0$ to the above problem. If it is different from the approximate solution you derived in the previous part, comment on the difference and whether it is significant.

2. Using WKB, derive connection formulas for

$$\epsilon^2 u'' - q(x)u = 0,$$

   where

$$\begin{aligned} q(x) &> 0 \text{ for } x > 0 \\ q(x) &< 0 \text{ for } x < 0 \\ \lim_{x \to 0^+} q(x) &= a^2 > 0 \\ \lim_{x \to 0^-} q(x) &= -b^2 < 0. \end{aligned}$$

   and give an expression for the leading order general solution in the limit of small ϵ.