

Math 228A
Homework 3
Due Tuesday, 12/02/08

1. Write a multigrid V-cycle code to solve the Poisson equation in two dimensions on the unit square with Dirichlet boundary conditions. Use full weighting for restriction, bilinear interpolation for prolongation, and red-black Gauss-Seidel for smoothing. Assume that the grid spacing is a power of two so that your coarsest grid consists of one interior point.

Suggestions: Begin by writing a simpler code, such as a two-grid solver in one spatial dimension. Then, modify this code to produce a 1D V-cycle code. If needed, write a 2D two-grid code. For a two-grid code, you can do a direct solve on the coarse grid, or simply use your smoother. Note that red-black Gauss-Seidel will give an exact solve in one cycle in one dimension, and so it is not the best smoother for testing in 1D.

Everyone should try to get the two-dimensional V-cycle code to work, but if you cannot, you may turn in one of these simplified codes for reduced credit. You should state what your code does, and use your code for the second problem of this assignment.

2. Numerically estimate the average convergence factor,

$$\left(\frac{\|e^{(k)}\|_\infty}{\|e^{(0)}\|_\infty} \right)^{1/k},$$

for different numbers of pre-smoothing steps, ν_1 , and post-smoothing steps, ν_2 , for $\nu = \nu_1 + \nu_2 \leq 3$. Be sure to use a small value of k because convergence may be reached very quickly. What test problem did you use? Report the results in a table, and discuss which choices of ν_1 and ν_2 give the most efficient solver.

3. Use your V-cycle code to solve

$$\Delta u = -\exp(-(x - 0.25)^2 - (y - 0.6)^2)$$

on the unit square $(0, 1) \times (0, 1)$ with homogeneous Dirichlet boundary conditions using a grid spacing of 2^{-7} . How many steps of pre and post-smoothing did you use? What tolerance did you use? How many cycles did it take to converge? Compare the amount of work needed to reach convergence with your solvers from Homework 2 taking into account how much work is involved in a V-cycle.