Math 228A
Homework 5
Due Friday, 12/10/10, 4:00 P.M.

Homework must be turned in to Arcade before the deadline. You may email him a pdf file or put a hard copy in his mailbox.

Exam week office hours:
Bob, Monday 12-1
Arcade, Tuesday & Wednesday 1:30-2:20

1. Write a program to solve the discrete Poisson equation on the unit square using preconditioned conjugate gradient. Set up a test problem and compare the number of iterations and efficiency of using (i) no preconditioning and (ii) SSOR preconditioning. Run your tests for different grid sizes. How does the number of iterations scale with the number of unknowns as the grid is refined?

   Note that there are two typos in the PCG algorithm in our textbook. See your class notes, another textbook, or the author's webpage for the correct algorithm.

   SSOR preconditioning Symmetric SOR (SSOR) consists of one forward sweep of SOR followed by one backward sweep of SOR. For the discrete Poisson equation, one step of SSOR is

$$u_{i,j}^{k+1/2} = \frac{\omega}{4}(u_{i-1,j}^{k+1/2} + u_{i,j-1}^{k+1/2} + u_{i+1,j}^{k} + u_{i,j+1}^{k} - h^2 f_{i,j}) + (1-\omega)u_{i,j}^{k}$$
$$u_{i,j}^{k+1} = \frac{\omega}{4}(u_{i-1,j}^{k+1/2} + u_{i,j-1}^{k+1/2} + u_{i+1,j}^{k+1} + u_{i,j+1}^{k+1} - h^2 f_{i,j}) + (1-\omega)u_{i,j}^{k+1/2}.$$

   It can be shown that one step of SSOR in matrix form is equivalent to

$$\frac{1}{\omega(2-\omega)}(D-\omega L)D^{-1}(D-\omega U)(\boldsymbol{u}^{k+1} - \boldsymbol{u}^{k}) = \boldsymbol{f},$$

   where $A = D - L - U$.

   For the constant coefficient problem, this suggests the preconditioner.

$$M = (D-\omega L)(D-\omega U).$$

   Note: If you are interested, experiment with incomplete Cholesky factorization preconditioning and multigrid preconditioning. Incomplete Cholesky preconditioning requires that you form the matrix. Vary the amount of fill (in MATLAB use `cholinc` and vary the drop tolerance). Obviously, a factorization with more elements results in fewer iterations of CG, but it is more expensive to compute and to apply the preconditioner. To use MG as a preconditioner, the product $M^{-1}r$ is computed by applying one V-cycle with zero initial guess with right hand side r. If the smoother is symmetric and the number of pre and post smoothing steps are the same, this preconditioner is symmetric positive definite and may be used with CG.