Exam 1

Math 17A: sections A01-A06

Fall 2007

Name Solutions

ID number ______

Circle your section number —

section	time	TA's name
A01	4:10	Colin
A02	5:10	Tasia
A03	6:10	Sonny
A04	7:10	Tasia
A05	7:10	Andy
A06	8:10	Tom

Directions

- Do not begin until instructed to do so.
- You may use pencils, pens, erasers, and calculators.
- Put away all books, notes, cell phones, or other electronic devices.
- Show all work for full credit.
- Keep your work as neat as possible. If we can't read it, we won't grade it.
- You must present your student ID when turning in your exam.

Point totals

problem	score
1	/15
2	/10
3	/15
4	/15
5	/10
6	/15
Total	/80

1. (5 points each, total 15) Evaluate the following limits if they exist. If they do not exist, state why. Show all work!

(a) $\lim_{x \to 1} g(x)$, where $g(x) = \begin{cases} x^2 - 3x + 2 & \text{for } x \neq 1 \\ 10 & \text{for } x = 1 \end{cases}$.

(b) $\lim_{x \to 0} \frac{|x|}{x}$

(c) $\lim_{x \to \infty} \left(6.3 + 4.5\,\mathrm{e}^{-x} \sin(2\pi x)\right)$

(a) $\lim_{x \to 1} g(x) = 1^2 - 3 \cdot 1 + 2 = 0$

(b) $\lim_{x \to 0} \frac{|x|}{x}$ does not exist because the left & right limits are not equal.

$\lim_{x \to 0^+} \frac{|x|}{x} = 1$ & $\lim_{x \to 0^-} \frac{|x|}{x} = -1$

(c) $-1 \leq \sin(2\pi x) \leq 1$

$-4.5 e^{-x} \leq 4.5 e^{-x} \sin(2\pi x) \leq 4.5 e^{-x}$

$6.3 - 4.5 e^{-x} \leq 6.3 + 4.5 e^{-x} \sin(2\pi x) \leq 6.3 + 4.5 e^{-x}$

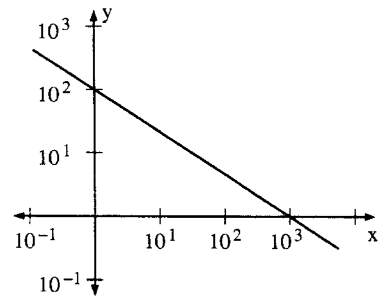
$\lim_{x \to \infty} (6.3 - 4.5 e^{-x}) = \lim_{x \to \infty} (6.3 + 4.5 e^{-x}) = 6.3$

By the sandwich theorem

$\lim_{x \to \infty} 6.3 + 4.5 e^{-x} \sin(2\pi x) = 6.3$

2. (a) (5 points) Let $Y = \log(y)$ and $X = \log(x)$. Based on the graph to the right, express Y as a function of X.

(b) (5 points) Write y as a function of x.



(a) $\text{Slope} = \dfrac{2-0}{0-3} = -\dfrac{2}{3}$

Y intercept $= 2$

$Y = -\frac{2}{3}X + 2$

(b) $Y = -\frac{2}{3}X + 2$

$\log(y) = -\frac{2}{3}\log(x) + 2$

$10^{\log(y)} = 10^{-\frac{2}{3}\log(x) + 2}$

$y = 10^{-\frac{2}{3}\log(x)} \cdot 10^2$

$y = 10^{\log(x^{-2/3})} \cdot 100$

$y = 100x^{-2/3}$

3. (5 points each, total 15) You solve for x in the equation $f(x) = 0$ using the bisection method, where f is some continuous function.

(a) Fill in the missing values in the table. Do not round.

a	$(a+b)/2$	b	$f(a)$	$f((a+b)/2)$	$f(b)$
5	7.5	10	3.21	-0.22	-0.87
5	6.25	7.5	3.21	0.89	-0.22
6.25	6.875	7.5	0.89	0.22	-0.22
6.875	7.1875	7.5	0.22	–	-0.22

(b) From the table, what is the approximate solution, and how accurate is it?

(c) Suppose you want to ensure that the error is less than 10^{-15}. How many steps of bisection would be needed?

(b) $X \approx 7.1875$

accurate to $(7.5 - 7.1875) = 0.3125$

(c) error $\leq 5 \cdot \left(\frac{1}{2}\right)^n$, where n is the number of steps

$$5 \cdot \left(\frac{1}{2}\right)^n = 10^{-15}$$

$$\left(\frac{1}{2}\right)^n = \frac{10^{-15}}{5}$$

$$\ln\left(\frac{1}{2}\right)^n = \ln\left(\frac{10^{-15}}{5}\right)$$

$$n \ln\left(\frac{1}{2}\right) = \ln\left(\frac{10^{-15}}{5}\right)$$

$$n = \frac{\ln\left(\frac{10^{-15}}{5}\right)}{\ln(1/2)}$$

$$n \approx 52.151$$

It would take 53 steps to ensure that the error $\leq 10^{-15}$

4. (5 points each, total 15) Calcium is commonly involved in intracellular signaling through modulating reaction rates. Suppose the reaction rate, R, of a process as a function of the calcium concentration, c, is

$$R = \frac{20\,c^4}{1000 + c^4}.$$

(a) To see how this rate behaves at very large calcium concentrations, compute $\lim_{c\to\infty} R$.

(b) Find the calcium concentration at which the reaction rate is one half of the value computed in part (a).

(c) In general, for reaction rates of the form

$$R = \frac{A\,c^n}{B^n + c^n},$$

state the physical significance of the constants A and B.

(a) $$\lim_{c\to\infty} R = \lim_{c\to\infty} \frac{20c^4}{1000+c^4} \cdot \frac{\frac{1}{c^4}}{\frac{1}{c^4}}$$

$$= \lim_{c\to\infty} \frac{20}{\frac{1000}{c^4}+1}$$

$$= \frac{20}{0+1} = 20$$

(b) $$\frac{1}{2}\cdot 20 = \frac{20\,c^4}{1000+c^4}$$

$$\frac{1}{2}(1000+c^4) = c^4$$

$$c^4 = 1000$$

$$c = 1000^{1/4} = 10^{3/4} \approx 5.62$$

(c) The reaction rate approaches A at very large calcium concentrations.

$$\frac{Ac^n}{B+c^n} \le A$$

A is the maximum reaction rate.

When $c = B$ the reaction rate is $A/2$. B is the concentration where the reaction rate is ½ of its maximum

5. (10 points) You measure the amount of radioactive material in a sample. You plot the amount of material remaining as a function of time (in days) on a semilog plot and find that the slope is -0.043. Find the half life of the material to the nearest day.

Let A be the amount of material.

$$\log(A) = -0.043t + b$$

$$A = 10^{b}\, 10^{-0.043t}$$

When $t=0$, $A(0) = 10^{b}$. $A(0)$ is the initial amount.

$$A = A(0)\, 10^{-0.043t}$$

Half life is the time when $A = \frac{1}{2}A(0)$.

$$\frac{1}{2}A(0) = A(0)\, 10^{-0.043t}$$

$$\frac{1}{2} = 10^{-0.043t}$$

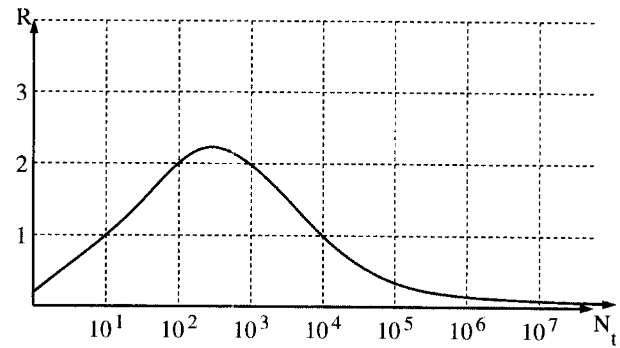
$$\ln\left(\frac{1}{2}\right) = -0.043t \ln(10)$$

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.043 \ln(10)}$$

$$t \approx 7.0$$

The half life is 7 days.

6. (5 points each, total 15) Suppose that the size of a population changes according to the recursion relation $N_{t+1} = R(N_t)\,N_t$, where $R(N_t)$ represents the reproductive rate and is a function of the population size. The graph of R is given below.



(a) What equation do the fixed points, N, satisfy? (hint: definition of a fixed point)

(b) Find all fixed points.

(c) Give a biological interpretation for each fixed point.

(a) $N = R(N)N$

(b) Either $N=0$ or $R=1$, which occurs at $N=10$ or 10^4.
The fixed points are $N=0, 10, 10^4$.

(c) $N=0$ – There is no population to reproduce.

$N=10$ – For populations below $N=10$, $R<1$. Just above 10, $R>1$. Populations below $N=10$ will go extinct & populations above $N=10$ will survive. Thus $N=10$ represents the minimum number of individuals to survive.

$N=10^4$ – For populations below $N=10^4$, $R>1$, & above $N=10^4$, $R<1$. Thus for $N<10^4$ (but bigger than 10) the population grows, & for populations above $N=10^4$ the population declines. The size $N=10^4$ is the maximum sustainable population, also called the carrying capacity.