Exam 2

Math 17A: sections A01-A06

Fall 2007

Name Solutions

ID number ______________________

Circle your section number —

section	time	TA's name
A01	4:10	Colin
A02	5:10	Tasia
A03	6:10	Sonny
A04	7:10	Tasia
A05	7:10	Andy
A06	8:10	Tom

Directions

- Do not begin until instructed to do so.
- You may use pencils, pens, erasers, and calculators.
- Put away all books, notes, cell phones, or other electronic devices.
- Show all work for full credit.
- Keep your work as neat as possible. If we can't read it, we won't grade it.
- You must present your student ID when turning in your exam.

Point totals

problem	1	2	3	4	5	6	7	total
score	/20	/10	/10	/10	/10	/15	/15	/90

1. (5 points each, total 20) Compute the derivative of the function with respect to the independent variable. Do no simplify your answer.

   (a) $w(t) = t^2 \cos(t)$

   (b) $g(x) = \dfrac{kx}{x^2 + a^2}$

   (c) $y = \sqrt{x^3 + 4x + 7}$

   (d) $f(x) = \ln(x^2 + 1)$

(a) $w'(t) = 2t\cos(t) - t^2 \sin(t)$

(b) $g'(x) = \dfrac{(x^2+a^2)k - kx(2x)}{(x^2+a^2)^2}$

(c) $y = (x^3+4x+7)^{1/2}$

$y' = \frac{1}{2}(x^3+4x+7)^{-1/2}(3x^2+4)$

(d) $f'(x) = \dfrac{1}{x^2+1}(2x)$

2. (a) (5 points) Find the equation of the line tangent to the curve $y = \sqrt{x}$ at the point $x = 100$.
(b) (5 points) Use the tangent line to approximate $\sqrt{103}$.

(a) Need to find the slope & ~~and~~ a point on the tangent line.

slope $\frac{dy}{dx} = \frac{1}{2} x^{-1/2}$

at $x = 100$ $\frac{dy}{dx} = \frac{1}{2}(100)^{-1/2} = \frac{1}{20}$

when $x = 100$, $y = 10$

$$y - 10 = \frac{1}{20}(x - 100)$$

$$y = \frac{1}{20}(x - 100) + 10$$

(b) Use equation of tangent line for $x = 103$

$$y = \frac{1}{20}(103 - 100) + 10$$

$$y = \frac{3}{20} + 10$$

$$y = \frac{203}{20}$$

$$\sqrt{103} \approx \frac{203}{20} = 10.15$$

3. (a) (5 points) For the function given below, identify where the function is increasing and decreasing. The constants a and b are both positive.

$$g(x) = bxe^{-ax^2}, \quad x \geq 0$$

(b) (5 points) What is the maximum value of g?

(a) To find where g is increasing & decreasing, we find where the derivative is positive & negative.

$$g'(x) = b e^{-ax^2} + bx e^{-ax^2}(-a2x)$$

$$g'(x) = b e^{-ax^2}(1 - 2ax^2)$$

$$g'(x) = 0 \quad \text{when} \quad 1 - 2ax^2 = 0$$

$$x = \pm\sqrt{\frac{1}{2a}}$$

Since domain is $x \geq 0$ the only place in the domain where $g'(x) = 0$ is $x = \sqrt{\frac{1}{2a}}$.

when $0 \leq x < \sqrt{\frac{1}{2a}}$, $g'(x) > 0$ (e.g. $g'(0) = be^{-ax^2} > 0$)

when $x > \sqrt{\frac{1}{2a}}$, $g'(x) < 0$ (for very large x $1 - 2ax^2 < 0$)

g is increasing $0 \leq x < \sqrt{\frac{1}{2a}}$

g is decreasing $x > \sqrt{\frac{1}{2a}}$

(b) The max is at $x = \sqrt{\frac{1}{2a}}$

$$g\left(\sqrt{\tfrac{1}{2a}}\right) = b\sqrt{\frac{1}{2a}}\, e^{-a\left(\frac{1}{2a}\right)} = \frac{be^{-1/2}}{\sqrt{2a}}$$

The max value of g is $\dfrac{be^{-1/2}}{\sqrt{2a}}$.

4. (10 points) Suppose you measure the volume of a sphere within an accuracy of 5% and use this value to compute the radius. How accurate is the radius that you compute? For a sphere of radius r, the surface area is $A = 4\pi r^2$ and the volume is $V = 4\pi r^3/3$.

Given $\frac{\Delta V}{V} = 0.05$ & want to find $\frac{\Delta r}{r}$.

$$\Delta V \approx V'(r)\,\Delta r = 4\pi r^2 \Delta r$$

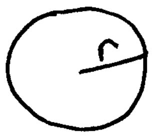
divide through by V

$$\frac{\Delta V}{V} \approx \frac{4\pi r^2 \Delta r}{V} = \frac{4\pi r^2 \Delta r}{\frac{4\pi r^3}{3}} = \frac{3\Delta r}{r}$$

$$\frac{\Delta r}{r} \approx \frac{1}{3}\frac{\Delta V}{V} = \frac{1}{3}(0.05) \approx 0.0167$$

The error in the radius is 1.67%.

5. (10 points) Oil from a ruptured tanker spreads in a circular pattern. If the radius of the circle increases at a constant rate of 1.5 feet per second, how fast is the area covered by oil increasing after 30 minutes from the beginning of the spill?



$A = \pi r^2$

given $\frac{dr}{dt} = 1.5 \frac{ft}{s}$ and we are looking for $\frac{dA}{dt}$.

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

Need to find r ~~to~~ at time 30 min.

~~[illegible]~~

Convert time into sec. so that time units are the same.

$$30 \text{ min} = 30 \cdot 60 s = 1800 s$$

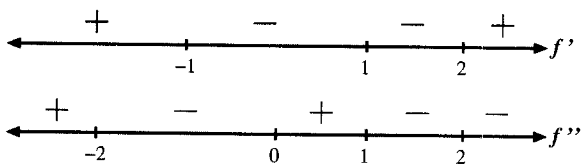
Because r is increasing at a constant rate when ~~[illegible]~~ t = 1800 s

$$r = 1.5 \frac{ft}{s} \cdot 1800 s = 2700 \text{ Ft}$$

$$\frac{dA}{dt} = 2\pi (2700 \text{ ft})(1.5 \frac{ft}{s}) = 8100\pi \frac{ft^2}{s} \approx 25{,}446.9 \frac{ft^2}{s}$$

The area is increasing at a rate of $8100\pi \frac{ft^2}{s}$ or $25{,}446.9 \frac{ft^2}{s}$.

6. (5 points each, total 15) Below you are given diagrams that show the sign of the first and second derivative for some function $f(x)$ where the domain of f is all real numbers. The points marked are where the respective derivative is zero or undefined.



(a) Identify the locations of local mins and local maxs?

(b) What are the x values of the inflection points?

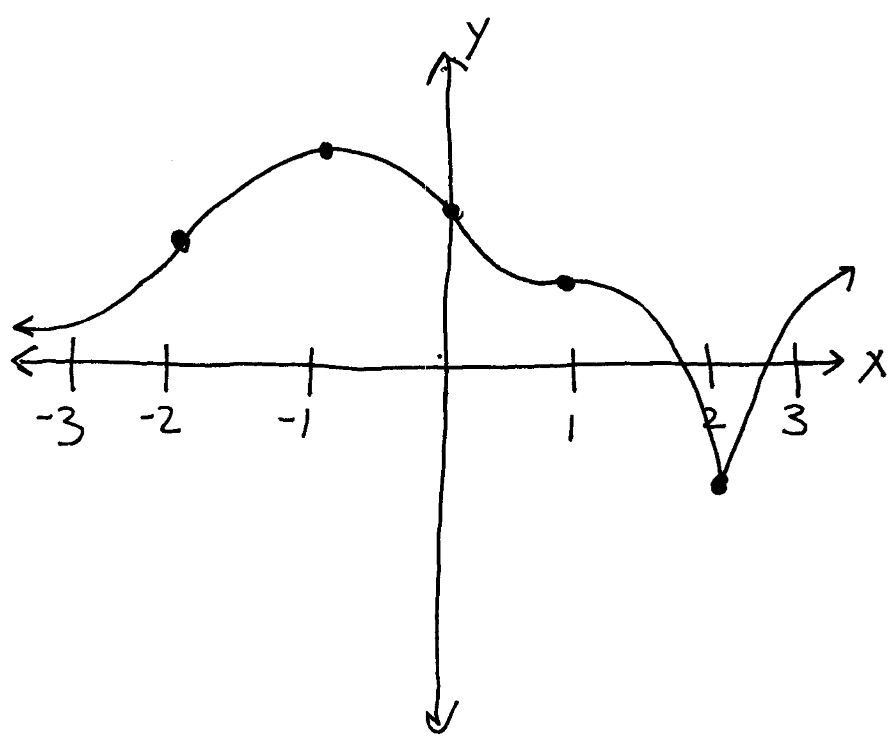
(c) Sketch a possible graph of the function.

(a) local max at $x=-1$

local min at $x=2$

(b) inflection points at $x=-2$ & $x=0$. & $x=1$.

(c)



Note: you could assume that the function is discontinuous at $x=1$ & $x=2$. If so, this changes your answers in parts (a) + (b).
It must be continuous at $x=-2,-1,0$ because it is differentiable at these points.

7. (5 points each, total 15) This question is about the meaning of a derivative. The formal definition of the derivative of a function $f(x)$ is $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ if the limit exists.

   (a) What is the meaning of the the derivative of a function $f(x)$ evaluated at a point x? (i.e. What does it tell you about the function?)

   (b) How does $\frac{f(x+h)-f(x)}{h}$ relate to your answer in part (a)?

   (c) Why does taking this limit as h goes to zero of the expression in part (b) give the derivative?

(a) The derivative is the slope of the tangent line.

OR

The derivative is the instantaneous rate of change of the function.

(b) $\frac{f(x+h)-f(x)}{h}$ is the slope of a secant line.

OR

$\frac{f(x+h)-f(x)}{h}$ is the average rate of change.

(c) As $h \to 0$ the two points on the curve through which the secant line passes come together. The secant line approaches the tangent line.

OR

The interval over which the average is taken gets smaller & smaller, and the average rate of change approaches the instantaneous rate of change.