Sample problems for second mid-term

1. **4.2.34** The vector $u = \{20, 30, 80, 10\}$ gives the number of receivers, CD players, speakers and cassette players on hand in a particular shop. The vector $v = \{100, 120, 80, 70\}$ gives the value of each receiver, CD player, speaker and cassette player. What is the total value of all goods on hand in that shop.

2. **4.2.35** A brokerage firm records the high and the low stock price of IBM stock each day. The information for a given business week is given by vectors h and l (what is the size of these vectors) giving the high and low values, respectively. What is the expression for the average daily value of the stock price of IBM over the 5 day period.

3. **4.2.T.13** Prove the parallelogram law:
$$|| \mathbf{u} + \mathbf{v} ||^2 + || \mathbf{u} - \mathbf{v} ||^2 = 2 || \mathbf{u} ||^2 + 2 || \mathbf{v} ||^2$$

4. **4.3.23/24** Determine whether the maps L_1, L_2 are linear transformations:
$$L_1 : \mathbb{R}^2 \to \mathbb{R}^2$$
$$L_1(x_1, x_2) \mapsto (x_1 + x_2 + 1, x_1 - x_2)$$
$$L_2 : \mathbb{R}^2 \to \mathbb{R}^2$$
$$L_2(x_1, x_2) \mapsto 5x_1 + 4x_2$$
.

5. **4, Supplementary Exercises, 12**. Let u_0 be a fixed vector in $\mathbb{R}^n$ and define the map $L : \mathbb{R}^n \to \mathbb{R}$ by $L(u) = u_0 \cdot u$. Show that L is a linear map. Side note, for every linear map $L : \mathbb{R}^n \to \mathbb{R}$, there is a unique vector $u_0 \in \mathbb{R}^n$ so that $L(u) = u \cdot u_0$.

6. **6.1.17** Let V be the set of all positive real numbers with operations
$$u \oplus v = uv$$
$$c \odot v = v^c.$$
Show that $(V, \oplus, \odot)$ is a vector space.

7. Let V be the space $\{(x_0, x_1)\}$ so that $x_0 > 0$ with $v \oplus u = v + u$ and $c \odot v = cv$ viewed as elements of $\mathbb{R}^2$. Is $(V, \oplus, \odot)$ a vector space.

8. Let V be the set of all polynomials of order n. Then is V, with the usual addition of polynomials and scalar multiplication a vector space.
9. **6.2.13** Let P be the set of all polynomials. Is P a vector space. Is P_n a subspace of P.
10. **6.2.18** Which of the following are subspaces of the space of all $n \times n$ matricies:

    a: The set of all $n \times n$ symmetric matricies (All $n \times n$ matricies A so that $A^t = A$).

    b: The set of all non-singular matricies.

    c: The set of all diagonal matricies.
11. **6.2.T.3** Show that the set of all solutions x of $Ax = b$ is not a subspace if $b \neq 0$.
12. Show that the set of all vectors $(x_1, x_2, \ldots, x_n)$ so that $x_1 + x_2 + \cdots + x + n = 0$ is a subspace. If $n = 2$, what does this subspace look like (sketch it).
13. **6.3.2** Which of the following sets vectors span $\mathbb{R}^3$.

    a:$\{(1, -1, 2), (0, 1, 1)\}$.

    b:$\{(1, 2, 1), (6, 3, 0), (4, -1, 2), (2, -5, 4)\}$

    c:$\{(2, 2, 3), (-1, -2, 1), (0, 1, 1)\}$

    d:$\{(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1)\}$.

    Which of the above are linearly independent. Which are a basis.
14. **6.3.4** Which of the following sets of polynomials space P_2.

    a: $\{t^2 + 1, t^2 + t, t + 1\}$

    b: $\{t^2 + 1, t - 1, t^2 - t\}$

    c: $\{t^2 + 2, 2t^2 - t + 1, t + 2, t^2 + t + 4\}$

    d: $\{t^2 + 2t - 1, t^2 - 1\}$

    Which are linearly independent. Which form a basis.

15. **6.3.7** Find a spanning set of the nullspace of

$$A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 2 & 3 & 6 & -2 \\ -2 & 1 & 2 & 2 \\ 0 & -2 & -4 & 0 \end{pmatrix}$$

Is this spanning set also a basis.

16. **6.4.12** Let $S\{v_1, v_2, v_3, v_4, v_5\}$ with

$$v_1 = (1, 1, 0, -1)$$

$$v_2 = (0, 1, 2, 1)$$

$$v_3 = (1, 0, 1, -1)$$

$$v_4 = (1, 1, -6, -3)$$

$$v_5 = (-1, -5, 1, 0)$$

Find a basis of $Span(S)$.

17. **6.4.17** Find a basis for the following subspaces of $\mathbb{R}^3$ and $\mathbb{R}^4$.

    a: All vectors (a, b, c) with $c = a + b$.

    b: All vectors (a, b, c, d) with $a + b + c + d = 0$.

18. Find the dimension of the subspace in the above problem.

19. **6.8.5**

20. **6.8.12**

21. **6.8.18**

22. **8.1.3**

23. **8.1.18**

24. **8.1.25/26**

25. For each of the subspaces found in the previous 3 problems, construct ortho-normal bases of the eigenspaces.

26. **8.1.T.6**

27. **8.1.T.9**

28. Diagonalize any of the above matricies that are diagonalizable.
29. **8.2.5**
30. **8.2.10**
31. **8.2.11/8.2.12/8.2.13**
32. **8.2.T.12**