Final Solutions

1) a) $\lim_{x\to -8} \frac{8+x}{2x+15} = 0$

b) $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{5-x} = \frac{\frac{\cancel{5-x}}{5x}}{\cancel{5-x}} = \lim_{x\to 5}\frac{1}{5x} = \frac{1}{25}$

c) $\lim_{x\to 0} \frac{2-\sqrt{4-x^2}}{x} \cdot \frac{2+\sqrt{4-x^2}}{2+\sqrt{4-x^2}} = \lim_{x\to 0} \frac{4-(4-x^2)}{x(2+\sqrt{4-x^2})} = \lim_{x\to 0} \frac{x^{\cancel{2}}}{\cancel{x}(\cancel{2+\sqrt{4-x^2}})} = 0.$

d) $\lim_{x\to 3^+} \frac{(x-4)(1-x)}{(x-5)(x-3)} \quad \frac{(-)(+)}{(-)\ (+)} = -\infty.$

2) a) Domain - possible values of independent variable (x) such that function $(f(x))$ is well defined $(\neq \pm\infty)$.

b) domain of $\sqrt{5-2x} \Rightarrow 5-2x \geq 0 \Rightarrow x \leq \frac{5}{2}.$

3) a) def. of derivative $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

b) $\lim_{h\to 0} \frac{\frac{4}{(x+h)^2}-\frac{4}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{4x^2-4(x+h)^2}{(x+h)^2(x)^2}}{h} = \lim_{h\to 0} \frac{\frac{-8xh-h^2}{(x+h)^2x^2}}{h}$

$= \lim_{h\to 0} \frac{4(-8x-h)}{(x+h)^2x^2} \cdot \frac{1}{h} = \frac{-8x}{x^4} = \frac{-8}{x^3}.$

4) a) $\left(\sqrt{\cos(5x+1)}\right)' = \frac{1}{2}\left(\cos(5x+1)\right)^{-1/2} \cdot \left(-\sin(5x+1)\cdot 5\right)$

b) $\left(\frac{(x+2)(x^3-5x+\pi)}{2x^3-5}\right)' = \frac{(2x^3-5)\left((x^3-5x+\pi)+(x+2)(3x^2-5)\right) - 6x^2(x+2)(x^3-5x+\pi)}{(2x^3-5)^2}$

5) a) differential $y = \frac{5x+3}{8} \qquad dy = \frac{5}{8}dx.$

b) when $x=1$, $dx = 0.02 \qquad dy = \frac{5}{8}(0.02) = \frac{1}{80}.$

6) spiderman: $\frac{dA}{dt} = \frac{3\pi}{5} \qquad A = \pi r^2 = 4\pi \Rightarrow r = 2.$

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{dA}{dt}\cdot\frac{1}{2\pi r} = \frac{3\pi}{5}\frac{1}{2\pi\cdot 2}$

$= \boxed{\frac{3}{20}}$

7) $y = \frac{x^2+2}{x^2-4}$ (like handout from class)

a) domain - $x \neq \pm 2$.

b) asymptotes v. asymptote $x = \pm 2$

h- asymptote $y = 1$

c) intervals $y' = \frac{2x(x^2-4) - 2x(x^2+2)}{(x^2-4)^2} = \frac{-12x}{(x^2-4)^2}$

c.p. at $x=0, \pm 2$

$(-\infty, -2)$ $(-2, 0)$ $(0, 2)$ $(2, \infty)$

$x=-3$ $x=-1$ $x=1$ $x=3$

$f'>0 \uparrow$ $f'>0 \uparrow$ $f'<0 \downarrow$ $f'<0 \downarrow$

d) 0 - a max

e) $y'' = \frac{-12(x^2-4)^2 - (-12x)(2(x^2-4))2x}{(x^2-4)^4} = \frac{36x^2+48}{(x^2-4)^3}$ $\Rightarrow x = \pm 2$ undefined.

$(-\infty, -2)$ $(-2, 2)$ $(2, \infty)$

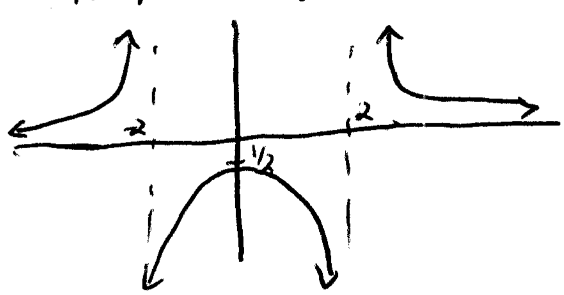
$x=-3$ $x=0$ $x=3$

$y''>0$ $y''<0$ $y''>0$

$\cup$ $\cap$ $\cup$

f) no inflection pt - $-2, 2$ not in domain!

g)



8) tangent line to $f(x) = \sqrt[3]{x^2} + 4$ at $(1, 5)$

$f'(x) = \frac{2}{3} x^{-1/3}$ at 1 $f'(1) = \frac{2}{3}$

$\Rightarrow y - 5 = \frac{2}{3}(x-1) \Rightarrow \boxed{y = \frac{2}{3}x + \frac{13}{3}}$

9) $x \cdot y = 192$

$x + 3y = S$

$$y = \frac{192}{x}$$

$$S = x + 3 \cdot \frac{192}{x} = x + \frac{576}{x}$$

$$S' = 1 - \frac{576}{x^2} = 0 \text{ when } x^2 = 576$$

$$\boxed{x = 24}$$

$$\Rightarrow y = \frac{192}{24} = \boxed{8.}$$

10) $f(x) = \cos(x) - \sin(x)$

$f'(x) = -\sin(x) - \cos(x)$

$f''(x) = -\cos(x) + \sin(x) = 0$ when $\cos(x) = \sin(x) \Rightarrow \cancel{x} = \frac{\pi}{4}, \frac{5\pi}{4}$

$\Rightarrow \left(0, \frac{\pi}{4}\right)$ $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ $\left(\frac{5\pi}{4}, 2\pi\right)$

$x = \frac{\pi}{6}$ $\quad x = \frac{\pi}{2}$ $\quad x = \frac{3\pi}{2}$

$f''(x) < 0$ $\quad f''(x) > 0$ $\quad f''(x) < 0$ $\quad \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4}$ inflection points.

$\cap$ $\quad \cup$ $\quad \cap$

11) $y = 12 - x^2$, closest to $(2, 4)$

$$d = \sqrt{(x-2)^2 + (12 - x^2 - 4)^2} = \sqrt{(x-2)^2 + (8 - x^2)^2}$$

$$d' = \frac{2(x-2) + 2(8 - x^2)(-2x)}{\sqrt{(x-2)^2 + (8 - x^2)^2}} = 0 \text{ when top} = 0$$

$\Rightarrow 4x^3 - 30x \cancel{+4} = 0$

? too hard - ~~if you did~~

there was a typo, should have been easier.

if you tried - you got 100%.

12) $f(x) = (x+1)^3(4-x)$

$f'(x) = 3(x+1)^2(4-x) - (x+1)^3 = (x+1)^2\left(3(4-x) - (x+1)\right) = (x+1)^2(11 - 4x)$

$= 0$ when $x = -1$ or $x = 11/4$.

$f''(x) = 6(x+1)(4-x) - 3(x+1)^2 - 3(x+1)^2 = 6(x+1)\left((4-x) - (x+1)\right)$

$= 6(x+1)(3 - 2x).$

$f''(-1) = 0$ — no info

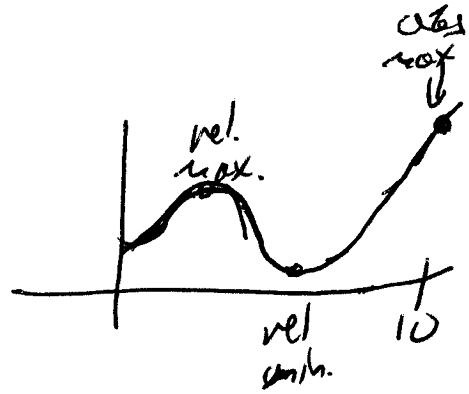
$f''(11/4) < 0 \Rightarrow 11/4$ max

13) $\left(x\sec(y) = 3y^2 + xy - 8\right)'$

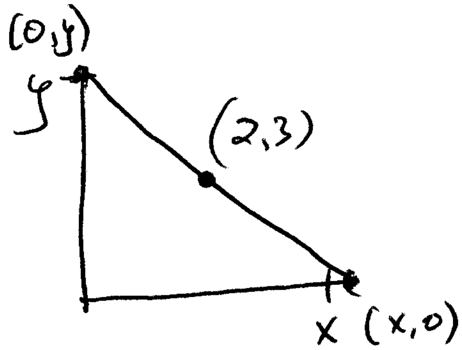
$$\sec(y) + x\sec(y)\tan(y)\cdot\frac{dy}{dx} = 6y\frac{dy}{dx} + y + x\frac{dy}{dx}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{y - \sec(y)}{x\sec(y)\tan(y) - 6y - x}}$$

14) Many right answers - one such answer



EC. 1)

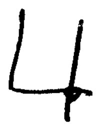


$$m = \frac{0-y}{\cancel{x}-0} = \frac{3-0}{2-x} \Rightarrow y = \frac{-3x}{2-x}$$

$$A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{-3x}{2-x}\right) = \frac{1}{2}\,\frac{-3x^2}{2-x}$$

$$A' = \frac{-3}{2}\left(\frac{2x(2-x) - (-1)x^2\,\cancel{(2x)}}{(2-x)^2}\right) = \frac{-3}{2}\left(\frac{-x^2+4x}{(2-x)^2}\right)$$

c.p. $x=2$, $x=0$, $x=4$ only one that makes sense

($x=2$  $-$ no Δ, $x=0$,  no Δ)

$\Rightarrow y = 6.$

2)
$$\frac{\left\{\left[-\sin\left(\frac{2x}{x+5}\right)\cdot\left(\frac{2(x+5)-2x}{(x+5)^2}\right)\cdot\left(3x\sqrt{\sec(x)}\right) + \cos\left(\frac{2x}{x+5}\right)\left(3\sqrt{\sec(x)} + \frac{3}{\sqrt{\sec(x)}}\cdot\sec(x)\tan(x)\right)\right]\cdot 15x + \cot^4(x^{1/3}) - \cos\left(\frac{2x}{x+5}\right)\left(3x\sqrt{\sec(x)}\right)\cdot\left(15 + 4\cot^3(x^{1/3})\cdot\left(-\csc^2(x^{1/3})\right)\cdot\frac{1}{3x^{2/3}}\right)\right\}}{\left(15x + \cot^4(x^{1/3})\right)^2}$$