Final Review Sol'n.

1) a) $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$ $\quad f'(x) = 0$ when $3x^2 = 12$

$\Rightarrow x = \pm 2.$

$f''(x) = 6x$ $\quad f''(x) = 0$ when $x = 0.$

$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$x = -3$	$x = 0$	$x = 3$
$f'(-3) > 0$	$f'(0) < 0$	$f'(3) > 0$
↑	↓	↑

$x = -2$ max $\quad x = 2$ min.

$(-\infty, 0)$	$(0, \infty)$
$x = -1$	$x = 1$
$f''(-1) < 0$	$f''(1) > 0.$
concave down ∩	concave up ∪

$\Rightarrow x = 0$ inflection pt.

b) $f(x) = x^{2/3} - 3$

$f'(x) = \frac{2}{3} x^{-1/3}$ $\quad f'(x)$ undefined when $x = 0.$

$(-\infty, 0)$	$(0, \infty)$
$x = -1$	$x = 1$
$f'(-1) = \frac{2}{3}\left(\frac{1}{\sqrt[3]{(-1)}}\right) < 0$	$f'(1) > 0$
↓	↑

$x = 0$ min. pt.

$\sqrt[3]{-1} = -1$ ✓ b/c $(-1)^3 = -1$...

$f''(x) = \frac{2}{3} \cdot \frac{-1}{3} x^{-4/3}$ $\quad -f''(x) = 0$ never, undefined at $x = 0.$

$(-\infty, 0)$	$(0, \infty)$
$x = -1$	$x = 1$
$f'(-1) < 0$ concave down	$f''(1) < 0$ concave down.

c) this question is hard - it won't be on the final!

$y = 2\sin(x) + \cos(2x)$

$y' = 2\cos(x) - 2\sin(2x) = 0$ when $\cos(x) = \sin(2x)$

$= 2\cos(x)\sin(x)$

$\Rightarrow x = \pi/2, 3\pi/2$ and when $\sin(x) = 1/2$

$\Rightarrow x = \pi/6, 5\pi/6$

$(0, \pi/6)$ $(\pi/6, \pi/2)$ $(\frac{\pi}{2}, 5\pi/6)$ $(\frac{5\pi}{6}, 3\pi/2)$ $(\frac{3\pi}{2}, 2\pi)$

$x = \frac{\pi}{12}$ $x = \pi/4$ $x = \frac{3\pi}{4}$ $x = \frac{5\pi}{4}$ $x = 7\pi/4$.

$f'(\frac{\pi}{12}) > 0$ $f'(\frac{\pi}{4}) < 0$ $f'(\frac{3\pi}{4}) > 0$ $f'(\frac{5\pi}{4}) < 0$ $f'(\frac{7\pi}{4}) > 0$

↑ ↓ ↑ ↓ ↑

$x = \pi/6$ max $x = \pi/2$ min. $x = 5\pi/6$ max. $x = 3\pi/2$ min.

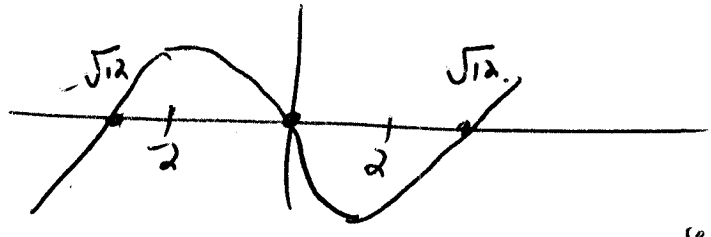
$y'' = -2\sin(x) - 4\cos(2x) = 0$ when $\sin(x) = -2\cos(2x)$.

$\Rightarrow \sin(x) = -2(1 - \sin^2(x)) = -2 + 2\sin^2(x)$

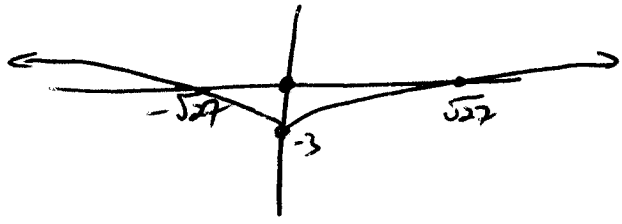
$2\sin^2(x) - \sin(x) - 2 = 0. \Rightarrow \sin(x) = \frac{1 \pm \sqrt{1+16}}{4}$

need to use quadratic here...

2) a) domain - all x, x-int at $x = 0$, $x = \pm\sqrt{12}$



b)

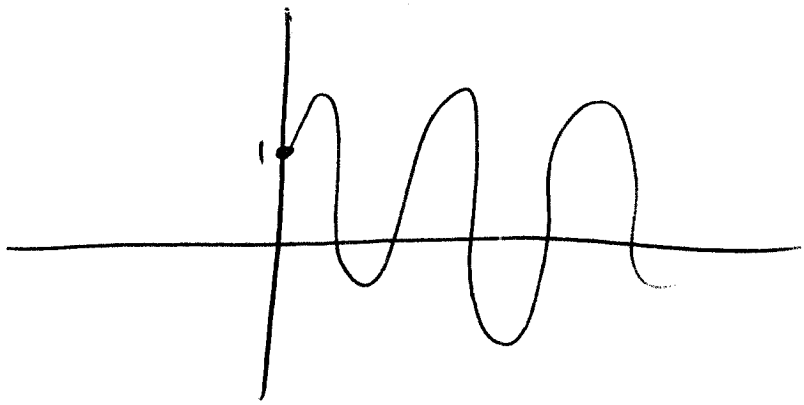


$y = x^{2/3} - 3$

y-int when $x^{2/3} = 3$

$x = \pm\sqrt{27}$

c)



Don't worry about this one...

3) $x - y = 50.$

$P = x \cdot y \qquad P = x \cdot (x - 50)$

$= x^2 - 50x$

$P' = 2x - 50 \qquad 2x - 50 = 0 \Rightarrow x = 25$

$\Rightarrow y = -25.$

$\Rightarrow P = -625$

if they were both > 0, take y as small as possible - almost 0 $\overset{0}{P}_{0} \Rightarrow x \cdot y \approx 0 \ldots$

4) $2x + 3y + 5 = 0 \Rightarrow y = \frac{2x - 5}{3}$

$d = \sqrt{(x - x_1)^2 + (y - y_1)^2}$

$= \sqrt{(x - (-1))^2 + \left(\frac{2x-5}{3} - (-2)\right)^2}$

$= \sqrt{(x+1)^2 + \left(\frac{2x-5}{3} + 2\right)^2}$

$d' = \frac{2(x+1) + 2\left(\frac{2x-5}{3}\right)\cdot\frac{2}{3}}{2\sqrt{(x+1)^2 + \left(\frac{2x-5}{3} + 2\right)^2}} = 0$ when $d' = 0$

(denominator can't $= 0$, no undefined points!)

$2x + 2 + \frac{8}{9}x - \frac{20}{9} = 0 \Rightarrow \frac{26}{9}x = \frac{2}{9} \qquad x = \frac{1}{13}.$

~~[illegible]~~ can check w/ 2nd derivative...

5) a) $dy = 2x\,dx$

b) $dy = \frac{5}{9}\,dx$

c) $dy = (24x^3 - 15x^2)\,dx$

6) $dy = 2x\,dx$

$= (2 \cdot 2)(0.01) = \boxed{0.04}$.