Midterm 1 Review, MAT 22B Hillel Raz

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The following is a study guide for MAT 22B, midterm 1. It is a list of topics covered that could be on the exam. The list of problems is longer than would be asked in a 50 minute exam, but it provides you with more sample problems so you can practice... Note that all examples from class, the book and homework problems should also be reviewed.

The exam will be closed book and notes and no calculator.

Ch. 1:

- Direction fields - know how to draw a direction field and how to interpret your graph. If there is a direct dependence on t, what happens to the graph, or to y in the limit as $t \to \infty$?

Examples:

1.$y' = 4y + 6$ 2.$y' = y(y+3)$

3.$y' = y + e^{-t}$ 4.$y' = y^2 - t^2/3$

- Know the different types of differential equations; ordinary DE, partial DE, linear vs. non-linear - and know how to tell the order of a DE.

Examples:

Classify the following DE's according to the above classifications and determine the order of each:

5.$y'' + 62(y')^4 + y''' = 5t$ 6.$y = \cos(t)\sin(t^2)y + y''$

7.$u_{xx} + u_{yy} + u_{xy} = u_{tt}$

- Be able to verify that a given function is a solution of a differential equation by plugging in. Similarly, solve for a parameter that makes a given function a solution of a DE.

Examples:

Verify that the following functions are solutions of the given DE,

8.$t^2y'' + 5ty' + 4y = 0;\quad t > 0;\quad y_1(t) = t^{-2},\quad y_2(t) = t^{-2}\ln t$

9.$y'' + y = 0;\quad y_1(t) = \cos(t),\quad y_2(t) = \cos(t) - \sin(t)$

Can you suggest another possible solution for number 9?

For $y = e^{rt}$, what values of r make the following DE true?

10.$y''' - 3y'' + 2y' = 0$

Ch. 2:

- Different methods of solving DE's - that is, integrating factor method and separation of variables. Know how to recognize when to use either.

For integrating factors,

I. Make sure equation is in correct form:

$$\frac{dy}{dt} + f(t)y = g(t).$$

II. Find $\int f(t)dt$.

III. Multiply equation (both sides!!!) by $e^{\int f(t)dt}$.

IV. Rewrite left hand side of the equation using reverse power rule (should become $(ye^{\int f(t)dt})'$).

V. Integrate and solve for y or whatever other variable is being used.

Note you can always check if your solution is correct by plugging it into the DE and seeing if its true (i.e. is the left hand side still equal to the right hand side?).

Examples:

Draw a direction field for the following then solve the DE's. Draw your solution on the direction field - it should 'flow' through the arrows (i.e. go in the same direction...).

11.$y' + y = 5\sin(2t), \quad y(0) = 1$ 12.$y' = \frac{x^2}{1+2y^3}, \quad y(1) = 4$

13.$y' = \frac{e^{-x}-e^x}{3+4y}, \quad y(0) = 1$ 14.$ty' - y = t^2e^{-t}, t > 0, \quad y(0) = 2$

- Models. There are lots of models. Know the different types we covered in class and in the homework (flow, interest, population models, etc.). Go with Steven Pon's method - **DUDES** :

Draw a picture. **U**se units. **D**eclare variables. **E**xpress derivative in terms of other stuff. **S**olve.

For population models, and other models, graphs are very beneficial. Make sure you understand how to draw and interpret the phase diagram, equilibrium points and stability. Of course, the equation needs to be autonomous - meaning there is only one variable written - i.e. $y' = f(y)$ not $y' = f(t, y)$ not $y' = f(t)$.

For flow problems, set up the DE remembering that the change = flow in - flow out. Usually you are asked to check concentration of something - hence you'll most likely need to divide the rate out part by the volume of the container as that affects the concentration (especially if that volume is constantly changing). A picture if of utmost importance here...

Examples - look at any in the book in section 2.3, 1-6 are flow problems for instance... Don't skip section 2.5 though! Try to set them up, that's the hardest part.

- Existence and uniqueness - know what the theorems say (2.4.1, 2.4.2, 2.8.1) and be able to explain in your own words what proving existence and uniqueness means.

Examples:

For the following, determine the interval in which a solution is certain to exist:

15.$(4 - t^2)y' + 2ty = 3t^2, \quad y(-3) = 1$ 16. $\ln(t)y' + 6y = \sin(t), \quad y(2) = 3$

- Euler's method. Know the method, how to set up and how to evaluate a few iterations (writing out the solutions without evaluating the actual numbers is ok - you won't be able to use a calculator on the exam).

Examples:

Use Euler's method with $h = 0.1$ to find approximate solutions for the given initial value problem for $t = 1 - 1.3$. Do you think the solutions are converging or diverging? (A direction

field might be helpful in answering the second question).
17. $y' = -4\sqrt{y^3}, \quad y(1) = 2$ 18.$y' = -ty + 0.1y^3, \quad y(0) = 1$

- Know the method of successive approximations. Be able to set it up and determine the formula for the nth iteration.

Examples:
Determine $\phi_n(t)$ for arbitrary n.
19. $y' = 2y + 2, \quad y(0) = 0$ 20.$y' = t^2y - 1, \quad y(0) = 0$

- Know the difference between a DE and a difference equation. Know how to solve a difference equation and find its equilibrium solutions.

Examples:
Solve the given difference equations in terms of the initial value y_0.
21. $y_{n+1} = -0.8y_n$ 22.$y_{n+1} = (-1)^{n+1}y_n$

Get enough sleep and eat a good meal the night before the exam!!!