Midterm 2 Study Guide, MAT 22B Winter '09

The midterm will cover chapter 3 and sections 6.1, 6.2.

A study guide for most of chapter 3 has been posted. Make sure you understand both the key concepts (linear independence, homogeneous vs. non-homogeneous, Wronskian and its use, difference between particular, complementary, general solution) as well as the different methods (solving second order homogeneous, non-homogeneous, how to use the Wronskian, how and when to use the methods of variation of parameters or undetermined coefficients, reduction of order, etc.).

One thing to emphasize that was not emphasized on most study guides, for second order homogeneous DE with repeated roots, the solution is **always** of the form

$$y(t) = C_1 e^{rt} + C_2\, t\, e^{rt}\,.$$

For complex roots, you solve for $r = \lambda \pm i\,\mu$ and then rewrite the solution as

$$y(t) = C_1 e^{\lambda\, t} \cos(\mu\, t) + C_2 e^{\lambda\, t} \sin(\mu\, t)\,.$$

Note that there is no i in the solution as written. It might come in via C_2 (or even C_1) but in most instances that we'll see, it won't be there.

Expect some conceptual questions like you did on the homeworks as this chapter has a lot of deep mathematical concepts. For practice problems, you can do more problems from the book, any problems I would provide would be similar in scope with different numbers. Review the concept homework problems.

Sections 3.7-3.8 will not be covered in depth in the exam. The main concepts in those questions are already covered in the rest of the material.

Sections 6.1-6.2:

These sections cover the Laplace Transform, an important mathematical tool. There are a few new definitions covered in this section. Know what piecewise-continuous function is and exponential growth means. You should also know what linearity means for an operator (think integral or derivative - Wronskian is a great example also, and of course, the Laplace transform).

This section deals mostly with integrating functions with against a kernel of e^{-st}. This means taking the integral of a function $f(t)$ multiplied with e^{-st} within the integral sign, i.e.

$$\int_0^\infty f(t)\, e^{-st} dt\,.$$

The function $f(t)$ is being integrated against is known as the kernel. Note that this integral is improper. If you need to, review how to integrate improperly... Also review further integration by parts and the method of partial fractions. The method of partial fractions is particularly important in doing the inverse Laplace transform. As a tip remember to simplify the problem as much as you can, and work from there.

You will **not** be required to memorize different Laplace transforms. You will be given the ones you need and possibly a few extra ones as well, just to keep you on your toes... There are plenty of problems in the book, do some extra ones for practice. The steps solving a problem using a Laplace transform are as follows:

1) Figure out the Laplace transform of each term in the equation (including the non-homogeneous one).

2) Rewrite the equation using the transform. Isolate $Y(s)$ and solve for it.

3) Work backwards to express in terms of possible Laplace transforms. Try to simplify as much as possible while looking at the chart on p. 319.

4) Rewrite solution using the inverse Laplace transform.

Good luck!!!