Study Guide for Final 21D Hillel Raz

The following is a study guide. It outlines the topics discussed in the class and provides sample problems. The emphasis here is on stuff that was taught after the second midterm. You are responsible for the material before and some questions on the final will regard that material either directly or will require you to use techniques taught in the first 2/3 of the course. To review that material review the class notes and examples, the homeworks, the first two study guides and the first two exams.
You will definitely be tested on the topics covered since the second exam. These include 16.3-16.8. The following is an outline reflecting the specific topics in those sections that you need to know.

- Conservative Fields and their properties. (16.3)
- The following defintions/concepts: connected, simply connected, potential functions, divergence, gradient, flux, curl, circulation.
- Know how to test if a field is conservative.
- Know how to find a potential function of a conservative field.
- Know what an exact form is and how to use it.
- Green's theorem - both versions - and how to use them. (16.4)
- Know the proofs done in class or in the homework (note that you are **not** responsible for the proofs done in the book unless they were done in class).
- Know how to find the surface area of a solid (hence know what the interpretation of $d\sigma$ is). (16.5)
- Know how to compute surface integrals and how to compute various properties involved with surface integrals such as flux, mass and moments.
- Be able to parametrize a surface using two variables and know how to use that parametrization to calculate the surface area and the various possible surface integrals. (16.6)
- Know how to compute the curl. (16.7)
- Stoke's theorem - and how to use it.
- The Divergence theorem - and how to use it. (16.8)
- Unifying all of the ideas from Green's, Stoke's and the Divergence theorem - that is understanding the connections (2-D to 3-D...).

Here are some sample problems. Note that there could very well be concept/definition questions and a proof that was done in class or the homeworks. These types of questions are not covered here.

1. Are the following fields conservative? If so, find their potential functions:
a) **F** = (y + z)**i** + (x + z)**j** + (x + z)**k**
b) $\mathbf{F} = e^{y+5z}(\mathbf{i} + x\mathbf{j} + 5x\mathbf{k})$

2. Can you evaluate the following integral using a shortcut? If yes, do so and explain why. If not, why not?

$$\int_{(1,2,1)}^{(2,1,1)} (2xlny - yz)dx + (\frac{x^2}{y} - xz)dy - xydz$$

What if you had to evaluate the same integral along the path from (1, 2, 1) to (7, 8, 9) and then from (7, 8, 9) to (2, 1, 1)?

3. Calculate the counterclockwise circulation and outward flux for the following field:
$\mathbf{F} = (x + y)\mathbf{i} - (x^2 + y^2)\mathbf{j}$ over the triangle bounded by y = 0, x = 1 and y = x.

4. Calculate the following integral $\oint_C y^2dx + x^2dy$ over the triangle x = 0, y = 0 and x + y = 1.

5. Find the flux of the field $\mathbf{F} = 4x\mathbf{i} + 4y\mathbf{j} + 2\mathbf{k}$ outward (away from the z-axis) through the surface

cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane z = 1.
(also problems like 34, 36 on p. 1175 are good).

6. Use parametrization to find the surface area of the given figure and then calculate the flux for $\mathbf{F} = z^2\mathbf{i} + x\mathbf{j} - 3z\mathbf{k}$ (away from the z-axis). The surface is given by the parabolic sylinder $z = 4 - y^2$ cut by the planes x = 0, x = 1 and z = 0.

7. Calculate the circulation for the force field given by $\mathbf{F} = y\mathbf{i} + xz\mathbf{j} + x\mathbf{k}$ over C where C is the boundary of the triangle cut from the plane x + y + z = 1 in the first octant (when viewed from above).
(also be able to do the same types of problems when given a parametrization of a surface - such as 13-18 on p. 1194).

8. Calculate the outward flux across the boundary region D where D is the solid region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 2$ while $\mathbf{F} = (5x^3 + 12xy^2)\mathbf{i} + (y^3 + e^z sinz)\mathbf{j} + (5z^3 + e^y cosz)\mathbf{k}$.