"Cheat Sheet"' for Taylor Series Or How I Learned To Stop Worrying and Love Power Series.

In order to succesfully master Taylor Series and 'graduate' from the 16 series, you will have to remember your definitions well.

Power Series: a power series is defined as a sum of: a sequence a_n times our variable x minus the center, c, to the nth power.

Mathematically, a power series is a series of the form $\sum_{n=0}^{\infty} a_n(x-c)^n$.

In Taylor Series problems, you will be given a function, $f(x)$, centered around a point (c) and you will be asked to find it's Taylor Series. In order to do this you will have to remember what the a_n are for power series.

The coefficients a_n are defined to be: $a_n = \dfrac{f^{(n)}(c)}{n!}$

The $f^{(n)}(c)$ is the nth derivative of the function $f(x)$ that you were given, evaluated at c. There should be no x term in your a_n! You will solve for the derivatives of $f(x)$ till you have figured out the pattern, so that you can write down what the nth derivative will be, and then you will evaluate **all** of the derivatives at c.
Again, there should be **no x in your a_n !!!**

Recipe, for slow cooking:

1) Given a function $f(x)$, write down it's derivatives in a column. You will usually have to solve for a few, possibly 5 or more, till you can figure out the pattern so that you can determine what the nth derivative will be. This brings us to step number 2.

2) Deduce the pattern of the derivatives. Write this out as $f^{(n)}(c)$. Note that you have not yet solved for a_n!!!

3) Write out a_n. That is, plug in from step 2) above into the following formula: $a_n = \dfrac{f^{(n)}(c)}{n!}$. Simplify when possible.

4) Plug in c. Simplify when possible.

5) You have figured out a_n. Now you just need to write out the Taylor Series as $\sum_{n=0}^{\infty} a_n(x-c)^n$. Remember to write c in for the $(x-c)$ term.

You are done! Hopefully you can now graduate from the UC Davis 16's!!!

Example: Find the Taylor series for $\frac{1}{1-x}$ centered at 0.

Step 1: Derivatives.

n = 0, $f^{(0)}(x)$which is the same as $f(x) =$

n = 1, $f^{(1)}(x) =$

n = 2, $f^{(2)}(x) =$

n = 3, $f^{(3)}(x) =$

n = 4, $f^{(1)}(x) =$

Step 2: At this point, if you can figure out the pattern for the nth derivative do it, otherwise take more derivatives till you can.

Formula for nth derivative: $f^{(n)}(x) =$

Step 3: Write out a_n. So plug the $f^{(n)}(x)$ in from above into the formula $a_n = \frac{f^{(n)}(c)}{n!} =$

Step 4: Plug in c and then simplify. Final a_n =

Step 5: Write the Taylor Series. $\sum_{n=0}^{\infty} a_n(x-c)^n$ =

Tip: You should always check to see if there is an issue at n = 0 (such as dividing by n - so when n = 0, dividing by 0), and if there is, what is a_0 equal to. You will have to write a_0 outside of the sum in such a case - many times though, a_0 will be simply 0 which will make things for you much simpler (example - Taylor Series of $log x$).