Key Midterm I 21D

1) a) $\int_0^{\pi/2}\int_0^{x} 2\cos(x+y)\,dy\,dx = \int_0^{\pi/2} 2\sin(x+y)\Big|_0^{x}\,dy = \int_0^{\pi/2} 2\sin 2x - 2\sin x\,dx$

$= -\cos 2x + 2\cos x\Big|_0^{\pi/2} = -(-1) + 0 + 1 - 2 = \boxed{0}$

b) $\int_0^{2}\int_{y/2}^{1} e^{x^2}\,dx\,dy = \int_0^{1}\int_0^{2x} e^{x^2}\,dy\,dx = \int_0^{1} e^{x^2} y\Big|_0^{2x} = \int_0^{1} 2xe^{x^2}\,dx$

$= e^{x^2}\Big|_0^{1} = \boxed{e-1}$

c) $\int_{\pi/4}^{\pi}\int_{\pi/x}^{2}\int_0^{\cos(xy)} x\,dz\,dy\,dx = \int_{\pi/4}^{\pi}\int_{\pi/x}^{2} x\cos xy\,dy\,dx = \int_{\pi/4}^{\pi} \sin xy\Big|_{\pi/x}^{2}\,dx$

$= \int_{\pi/4}^{\pi} \sin 2x - \sin\pi\,dx = -\frac{\cos 2x}{2}\Big|_{\pi/4}^{\pi} = \frac{-1}{2} + 0 = \boxed{-\frac{1}{2}}$

2) a) $x \le y \le 3x$
$0 \le x \le 2$

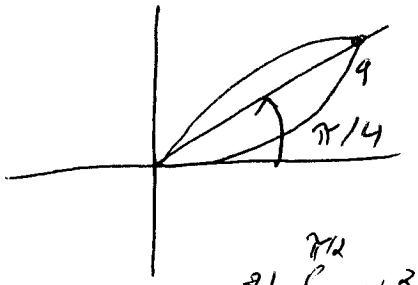
b) $\frac{y}{3} \le x \le 2$
$2 \le y \le 6$
or
$\frac{y}{3} \le x \le y$
$0 \le y \le 2$

c) $0 \le r \le 2\sec\theta$
$\pi/4 \le \theta \le \tan^{-1}(3)$

3) a) $M = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2} z\,dz\,dy\,dx$ (other orders ok)
↑ $\sigma(x,y,z)$

b) $\bar{x} = \frac{M_{yz}}{M} = \dfrac{\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2} xz\,dz\,dy\,dx}{M}$

4)



$$\int_0^{\pi/2}\int_0^{9\sin 2\theta} r\,dr\,d\theta = \int_0^{\pi/2} \frac{r^2}{2}\Big|_0^{9\sin 2\theta} d\theta =$$

$$= \frac{81}{2}\int_0^{\pi/2} \sin^2 2\theta\, d\theta = \frac{81}{2}\int_0^{\pi/2} \frac{1}{2} - \frac{\cos 4\theta}{2}\, d\theta = \frac{81}{2}\left(\frac{1}{2}\theta - \frac{\sin 4\theta}{8}\right)\Big|_0^{\pi/2}$$

$$= \boxed{\frac{81\cdot\pi}{8}}$$

5) a) $-\sqrt{z-y^2} \le x \le \sqrt{z-y^2}$

$-\sqrt{z} \le y \le \sqrt{z}$

$0 \le z \le 4$

6) $x^2+y^2 \le z \le 4$

$-\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$

$-2 \le y \le 2$

6) $I_L = \int_0^2 \int_y^{3/2y} (x^2+y)(2-y)^2\, dx\, dy$ or $\int_0^2 \int_x^{2/3x} (x^2+y)(2-y)^2\, dy\, dx$

$+$

$\int_2^3 \int_x^2 (x^2+y)(2-y)^2\, dy\, dx.$

7) $\frac{1}{2\pi}\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \sqrt{x^2+y^2}\, dx\, dy = \frac{1}{4\pi}\int_0^{2\pi}\int_0^2 r\cdot r\, dr\, d\theta = \frac{1}{4\pi}\frac{r^3}{3}\Big|_0^2 \cdot 2\pi = \frac{2\pi}{4\pi}\frac{8}{3}$

$= \boxed{\frac{4}{3}}$

E.C. need points where domain is positive $\Rightarrow x^2+y^2+z^2 \le 1$ - solid sphere of radius 1 centered at origin.