1) $(3y^2 + x^2y = C)' \Rightarrow 6yy' + 2xy + x^2y' = 0$

$\Rightarrow 2xy + y'(6y + x^2) = 0.$ ✓

$y = -1, x = 2 \Rightarrow 3(-1)^2 + (2)^2(-1) = C$

$\Rightarrow 3 - 4 = C \Rightarrow C = -1$

$\Rightarrow \boxed{3y^2 + x^2y = -1}$

2) a) $\frac{dy}{dx} = \frac{e^x}{4y}$ — Sep. of Var.

$\Rightarrow \int 4y\,dy = \int e^x\,dx$

$\Rightarrow \boxed{2y^2 = e^x + C}$

b) $xy' - 4xe^{x^4} = -3y; \quad x > 0$

1st, write in standard form (1st order-linear DE)

$\Rightarrow y' - \frac{4xe^{x^4}}{x} + \frac{3y}{x} = 0 \Rightarrow y' + \frac{3}{x}\cdot y = 4e^{x^4}.$

$\Rightarrow P(x) = \frac{3}{x}, \quad Q(x) = 4e^{x^4}$

$\Rightarrow U(x) = e^{\int P(x)dx} = e^{\int \frac{3}{x}dx} = e^{3\ln x} = e^{\ln x^3} = x^3$

this is why $x > 0$!

$y = \frac{1}{x^3}\left(\int 4e^{x^4}\cdot x^3\,dx\right)$ U-substitution,

$u = x^4, \quad du = 4x^3\,dx$

$\Rightarrow y = \frac{1}{x^3}\left(\int e^u\,du\right) = \frac{1}{x^3}\left(e^u + C\right) = \boxed{\frac{1}{x^3}e^{x^4} + \frac{C}{x^3}}$

c) $y' + y = 6e^x$, standard form ✓

$P(x) = 1, \quad Q(x) = 6e^x.$

$\Rightarrow U(x) = e^{\int dx} = e^x.$

$\Rightarrow y = \frac{1}{e^x}\left(\int 6e^x\cdot e^x\,dx\right) = \frac{1}{e^x}\left(\int 6e^{2x}\,dx\right) = \frac{1}{e^x}\left(\frac{6e^{2x}}{2} + C\right)$

$y = \boxed{3e^x + Ce^{-x}}$

d) $y' = \frac{x}{y} - \frac{x}{1+y}$ $\Rightarrow$ Sep. of Var.

1st, common denominator:

$$\frac{(1+y)x - y\cdot x}{y(1+y)} = \frac{x}{y(1+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y(1+y)} \Rightarrow (y+1)y\,dy = x\,dx$$

$$\Rightarrow \int (y^2+y)\,dy = \int x\,dx$$

$$\boxed{\frac{y^3}{3} + \frac{y^2}{2} = \frac{x^2}{2} + C}$$

Second 2) (sorry...)

Let Q be the concentration of the · in the tank.

$$\Rightarrow \underbrace{\frac{dQ}{dt}}_{\text{rate of change of concentration of}} = -\frac{10Q}{\underbrace{50}_{\text{half full!}}} + 15$$

$\leftarrow$ rate in (1.5 lbs/gallon, 10 gallons a minute)

rate out - hence the negative sign.

$\Rightarrow$ in standard form: $\frac{dQ}{dt} + \frac{1}{5}Q = 15$, $\quad P(t) = 1/5$
$Q(t) = 15$
(different Q - this one is a function of t...

$$\Rightarrow U(t) = e^{\int \frac{1}{5} dt} = e^{1/5 t}$$

$$\Rightarrow Q = \frac{1}{e^{1/5 t}}\left(\int 15 e^{1/5 t}\,dt\right) = \frac{1}{e^{1/5 t}}\left(\frac{15}{1/5} e^{1/5 t} + C\right)$$

$$= 75 + Ce^{-1/5 t}$$

At $t=0$, $Q=0$! (tank is full of water!) $\Rightarrow 0 = 75 + C$, $\Rightarrow C = -75$

$$\Rightarrow Q = 75 - 75e^{-1/5 t}$$

So at $t = 30$ min, $Q = 75 - 75e^{-1/5 \cdot 30} = \boxed{75 - 75e^{-6}}$

3) Let M = Population of monkeys

$\Rightarrow \frac{dM}{dt} = \underbrace{k \cdot}_{\text{proportional to}} (M)^{1/2}$ ← square root.

$\Rightarrow \frac{dM}{M^{1/2}} = k\,dt \Rightarrow \int M^{-1/2}\,dM = \int k\,dt$

$\Rightarrow 2M^{1/2} = kt + C$

$\Rightarrow M = \left(\frac{kt+C}{2}\right)^2.$

$t=0,\ M=100 \Rightarrow 100 = \left(\frac{C}{2}\right)^2 \Rightarrow C = \sqrt{100}\cdot 2$

$\Rightarrow C = 20$

$\Rightarrow M = \left(\frac{kt+20}{2}\right)^2$

$t=2,\ M=400 \Rightarrow 400 = \left(\frac{2k+20}{2}\right)^2$

$\Rightarrow 20 = \frac{2k+20}{2} \Rightarrow 40-20 = 2k \Rightarrow k = 10.$

$\Rightarrow M = \left(\frac{10t+20}{2}\right)^2$

So at $t=5$, $M = \left(\frac{10\cdot 5+20}{2}\right)^2 = \left(\frac{70}{2}\right)^2 = \boxed{35^2}$ ← ok like this.

4) (24) Gompertz growth model $\frac{dy}{dt} = ky\ln\left(\frac{L}{y}\right)$

$L = 400$ (maximum sustainable popl.)

$\Rightarrow \frac{dy}{y\ln\left(\frac{400}{y}\right)} = k\,dt$, u subst., $u = \ln\left(\frac{400}{y}\right)$

$du = -\frac{1}{y}\,dy$

$\Rightarrow \frac{du}{u} = -k\,dt \Rightarrow \int \frac{du}{u} = -\int k\,dt$

$\Rightarrow \ln u = -kt + C \Rightarrow \ln\left(\ln\left(\frac{400}{y}\right)\right) = -kt + C$

$\Rightarrow e^{-kt+C} = \ln\left(\frac{400}{y}\right) \Rightarrow e^{Ce^{-kt}} = \frac{400}{y}$

$$y = 400e^{-ce^{-kt}}.$$

$$t=0,\ y=30 \Rightarrow 30 = 400e^{-c} \Rightarrow c = -\ln\left(\frac{30}{400}\right)$$

$$t=1,\ y=90 \Rightarrow 90 = 400e^{\ln\left(\frac{30}{400}\right)e^{-k\cdot 1}}$$

$$\Rightarrow \ln\left(\frac{90}{400}\right) = \ln\left(\frac{30}{400}\right)e^{-k} \Rightarrow \frac{\ln\left(\frac{90}{400}\right)}{\ln\left(\frac{30}{400}\right)} = e^{-k}$$

$$\Rightarrow k = -\ln\left(\frac{\ln\left(\frac{90}{400}\right)}{\ln\left(\frac{30}{400}\right)}\right)$$ or like this.

so at $t=3$, $\boxed{y = 400e^{+\ln\left(\frac{30}{400}\right)e^{\ln\left(\frac{\ln\left(\frac{90}{400}\right)}{\ln\left(\frac{30}{400}\right)}\right)\cdot 3}}}$

5) $4x^2+4y^2+4z^2+8x+12z+7=0$

$$\Rightarrow x^2+y^2+z^2+2x+3z = -7/4$$

$$\Rightarrow (x^2+2x+\quad) + y^2 + (z^2+3z+\quad) = \frac{-7}{4}$$

$$\Rightarrow (x^2+2x+1) + y^2 + \left(z^2+3z+\left(\frac{3}{2}\right)^2\right) = \frac{-7}{4}+1+\left(\frac{3}{2}\right)^2$$

$$\Rightarrow (x+1)^2 + y^2 + \left(z+\frac{3}{2}\right)^2 = 3/2$$

$\Rightarrow$ $\boxed{\text{center at } \left(-1, 0, \frac{-3}{2}\right),\ \text{radius} = \sqrt{\frac{3}{2}}}$

6) use distance formula to find the diameter:

$$\sqrt{(2-7)^2+(4--3)^2+(5-0)^2} = \sqrt{5^2+7^2+5^2} = \sqrt{99}$$

$$\Rightarrow \text{radius} = \frac{\sqrt{99}}{2} = \frac{3\sqrt{11}}{2}.$$

For center, use midpoint formula: $\left(\frac{2+7}{2}, \frac{4-3}{2}, \frac{5+0}{2}\right) = \left(\frac{9}{2}, \frac{1}{2}, \frac{5}{2}\right)$

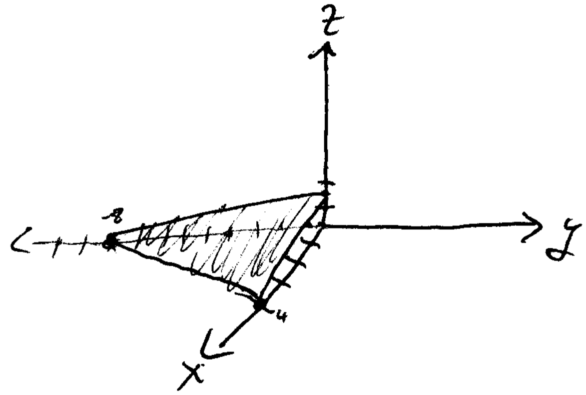
$\Rightarrow$ equation of the sphere:

$$\boxed{\left(x-\frac{9}{2}\right)^2+\left(y-\frac{1}{2}\right)^2+\left(z-\frac{5}{2}\right)^2 = \left(\frac{3\sqrt{11}}{2}\right)^2}$$

7) $2x - y + 5z = 8$, so x int: $2x = 8$, $x = 4$

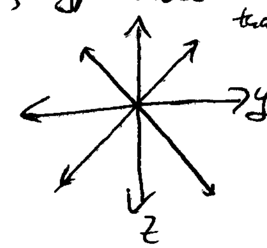
y int.: $-y = 8$

z int.: $5z = 8$, $z = 8/5$



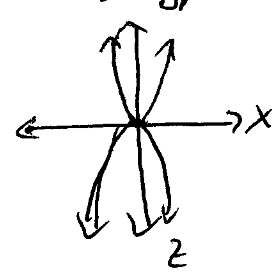
8) a) $z^2 = 9x^2 + y^2$.

Traces: $x = 0 \Rightarrow z^2 = y^2$, hyperbola ("straight lines" though)

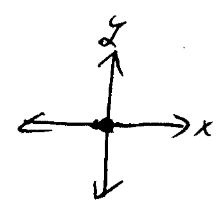


$y = 0 \Rightarrow z^2 = 9x^2$, hyperbola

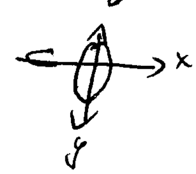
(z big ~~...~~, x smaller)



$z = 0 \Rightarrow 9x^2 + y^2 = 0$ - point at 0.



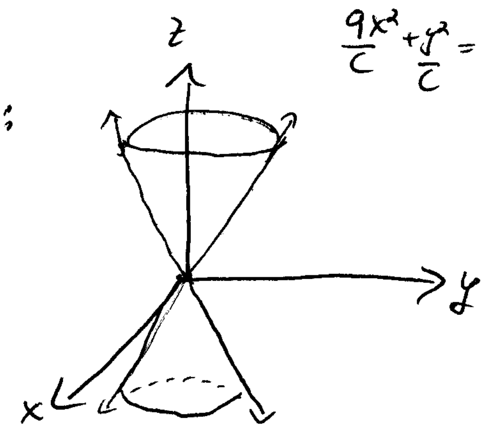
$z = C \Rightarrow 9x^2 + y^2 = C$, ellipse



$\frac{9x^2}{C} + \frac{y^2}{C} = 1 \Rightarrow$ longer on the y-axis.

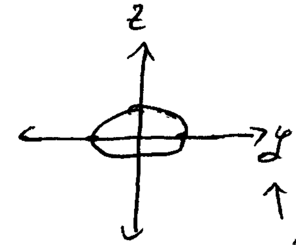
Putting everything together:

elliptic cone opening on the z-axis.



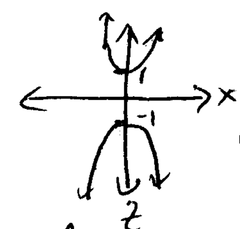
b) $-2x^2+y^2+5z^2=1$

Traces: $x=0 \Rightarrow y^2+5z^2=1$, ellipse



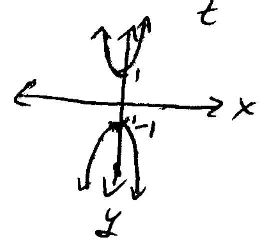
↑ larger on y-axis!

$y=0 \Rightarrow -2x^2+5z^2=1$, hyperbola

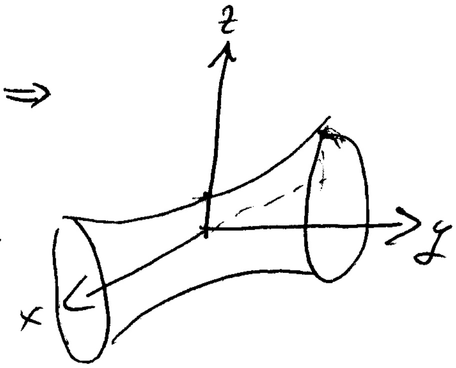


(z has positive! coefficient)

$z=0 \Rightarrow -2x^2+y^2=1$, hyperbola



putting traces together $\Rightarrow$

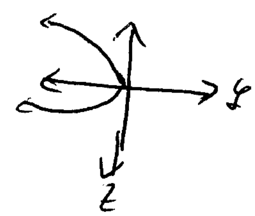


Hyperboloid of one sheet.

opens on the x-axis.

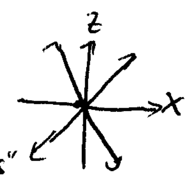
c) $2y=x^2-z^2$ Traces:

$x=0 \Rightarrow 2y=-z^2$, parabola

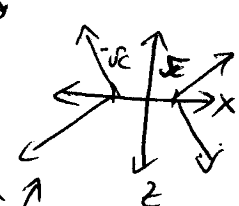


(negative coefficient on z!)

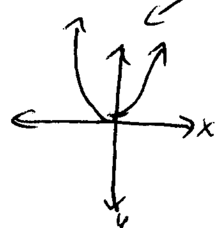
$y=0 \Rightarrow 0=x^2-z^2$, hyperbola "straight lines"



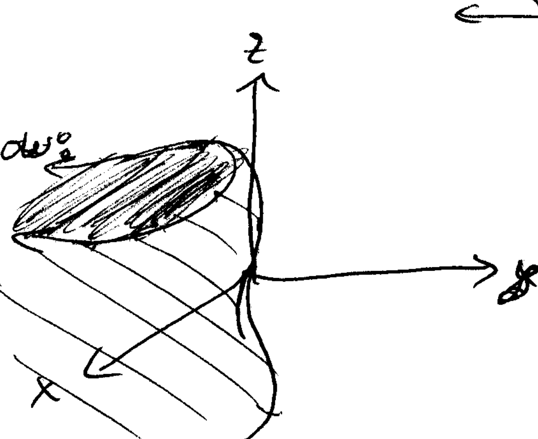
, $y=c \Rightarrow x^2-z^2=c$



$z=0 \Rightarrow 2y=x^2$, parabola



all together now:



Hyperbolic Paraboloid.

opens on the $-y$ axis.

Good luck!