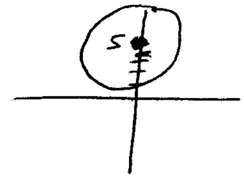
Mid term 1   16A

1) $\sqrt{(1--3)+(5-2)^2} = \sqrt{4^2+3^2} = 5$

2) $x^2-10y+y^2+9=0 \Rightarrow x^2+y^2-10y+25 = -9+25$ center (0,5) radius 4

$x^2+(y-5)^2 = 4^2$



3) a) $f(x)=\frac{5x}{3x+7}$ $3x+7 \neq 0 \Rightarrow x \neq -\frac{7}{3}$ domain all real #'s except for $x=-\frac{7}{3}$

b) $f(x) = 9x+\sqrt{x^2-9}$ ≥ 0

$\Rightarrow x^2-9 \geq 0$

$(x+3)(x-3) \geq 0 \Rightarrow x \geq 3$ & $x \leq -3$.

4) $f(x) = \frac{x^2-4}{x^2-x-6} = \frac{(\cancel{x+2})(x-2)}{(x-3)(\cancel{x+2})} \Rightarrow$ v asympt. $x=3$

h. asympt. as $x \to \infty$, $y=1$.

5) a) $\lim_{x\to -3} \frac{3}{x+2} = \frac{3}{-1} = -3.$

b) $\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x\to 9} \frac{(\cancel{x-9})(\sqrt{x}+3)}{\cancel{x-9}} = \sqrt{9}+3 = 6.$

c) $\lim_{x\to 3^-} \frac{x-6}{(x+5)(x-3)}$ check close to 3 but below (from the left)

$\frac{\text{neg.}}{(\text{positive})(\text{neg})} \Rightarrow \to +\infty.$

d) $\lim_{x\to\infty} \frac{x^2-12{,}008x+9\pi}{4x^2-22x+8x^3}$ — biggest degree $\to 0.$

6) a) $F(g(x)) = \frac{\frac{x}{1-x}-3}{\frac{x}{1-x}+4} = \frac{\frac{x-3(1-x)}{\cancel{(1-x)}}}{\frac{x+4(1-x)}{\cancel{(1-x)}}} = \frac{x-3+3x}{x+4-4x} = \boxed{\frac{4x-3}{4-3x}}$

b) $g(x)^{-1} \Rightarrow x = \frac{y}{1-y} \Rightarrow x(1-y) = y$

$x - xy = y \Rightarrow x = y+xy$

$= y(1+x)$

$\Rightarrow \boxed{y = \frac{x}{1+x}}$

7) From class/book:

1. f needs to be defined at the point
2. the limit at the point needs to exist (i.e. $\lim_{x\to c} f(x)$ exists)
3. the limit at the point must equal the value of the function at that point,

$\lim_{x\to c} f(x) = f(c)$.

8) a) def. of derivative: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

b) $\lim_{h\to 0} \frac{\sqrt{x-2+h}-\sqrt{x-2}}{h} \cdot \frac{\sqrt{x-2+h}+\sqrt{x-2}}{\sqrt{x-2+h}+\sqrt{x-2}} = \lim_{h\to 0} \frac{\cancel{x}-\cancel{2}+h-(\cancel{x}-\cancel{2})}{h(\sqrt{x-2+h}+\sqrt{x-2})}$

$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x-2+h}+\sqrt{x-2})} = \lim_{h\to 0} \frac{1}{\sqrt{x-2+h}+\sqrt{x-2}} = \frac{1}{\sqrt{x-2+0}+\sqrt{x-2}} = \boxed{\frac{1}{2\sqrt{x-2}}}$

9) a) all constants $\to$ 0 $= 0$.

b) $7x^{4/3} = 7 \cdot \frac{4}{3} x^{4/3 - 1} = \frac{28}{3} x^{1/3}$.

c) $15x^{14} - 14$.

10) $f'(x) = \frac{10}{4}x^9 - \frac{3}{2}\frac{1}{\sqrt{x}}$ at $x = 1 \Rightarrow f'(1) = \frac{10}{4} - \frac{3}{2} = 1$.

$\Rightarrow y - \frac{19}{4} = x - 1 \Rightarrow \boxed{y = x + \frac{15}{4}}$

Bonus:

$\frac{x^3-1}{x^2-1} = \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} \Rightarrow$ at 1, $\Rightarrow \frac{1+1+1}{1+1} = \frac{3}{2}$.

cont. at 1, not continuous at -1 !.