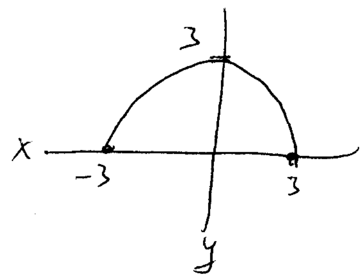
midterm 2 Solutions:

1) done in class + in book.

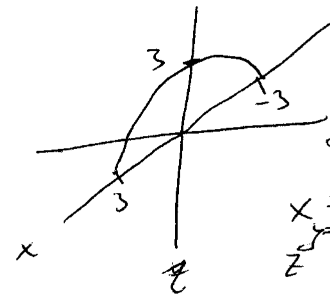
2) $$\int_0^{2\pi}\int_0^{1}\int_0^{r\sin\theta+2} (r\cos\theta)^2 z\,(r^2)\,dz\,r\,dr\,d\theta$$

3)



$x = 3\cos t \qquad 0\le t\le\pi$
$y = 3\sin t$
$z = 0 \qquad \int \vec F\cdot d\vec r =$

$$\int_0^{\pi} (3\cos t)(-3\sin t) + 0\cdot dt$$
$$= -9\int_0^{\pi}\cos t\sin t = \frac{9\cos^2 t}{2}\Big|_0^{\pi}$$
$$= \frac{9(-1)^2}{2} - \frac{9(1)^2}{2} = \boxed{0}$$



$x = 3\cos t$
$y=0$
$z = 3\sin t$
$0\le t\le\pi$

$$\int \vec F\cdot d\vec r = \int_0^{\pi} (3\cos t - 3\sin t)(-3\sin t) + (3\cos t)(3\cos t)\,dt$$
$$= \int_0^{\pi} -9\cos t\sin t + 9(\cos^2 t + \sin^2 t)\,dt$$
$$= 9t + \frac{\cos^2 t}{2}\Big|_0^{\pi} = \boxed{9\pi}$$

4) in class.

5) Note that $z = w - v \Rightarrow y = v + x - z = v + x - (w - v) = 2v + x - w.$

also $y = u - 2x + z = u - 2x + w - v.$

$$\Rightarrow 3x = 2w + u - 3v \Rightarrow \boxed{x = \frac{2w}{3} + \frac{u}{3} - v}$$

$$\Rightarrow y = 2v + \left(\frac{2w}{3} + \frac{u}{3} - v\right) - w = \boxed{v - \frac{w}{3} + \frac{u}{3}}$$

$$J(u,v,w) = \begin{vmatrix} \frac13 & -1 & \frac23 \\ \frac13 & 1 & -\frac13 \\ 0 & -1 & 1 \end{vmatrix} = -(-1)\left(\frac13\left(\frac{-1}{3}\right) - \frac23\left(\frac13\right)\right) + 1\left(\frac13 - \left(-\frac13\right)\right)$$

$+ \quad - \quad +$

$$= -\frac13 + \frac23 = \frac13$$

bounds $2x + y - z = u = 2$, $2x + y - z = 8 = u$, $z + y - x = v = 7$

$y - x + z = 6 = v$, $-x + 2z + y = w = 3$, $y - x + 2z = w = 5.$

$$\Rightarrow \frac{1}{3}\int_2^8\int_6^7\int_3^5 (w-v)u\, e^{u^2}\, dw\, dv\, du = \left(-\frac{5}{2}\right)\frac{1}{3}\left(e^{64}-e^{4}\right)$$

6) a) $f(x,y,z) = x^3e^{2y} - \frac{3y}{z-4}$ $\quad \nabla f = 3x^2e^{2y}\vec{i} + \left(2x^3e^{2y} - \frac{3}{z-4}\right)\vec{j} + \frac{3y}{(z-4)^2}\vec{k}$

so at $(6,0,5) \Rightarrow 108\vec{i} + \underbrace{(2\cdot 6^3 - 3)}_{=429}\vec{j}$

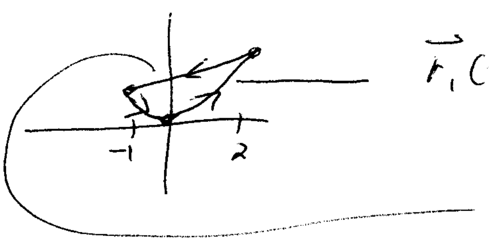
b) vector in direction in which f undergoes the greatest rate of ~~change~~ increase and the vector has magnitude equal to that rate of increase.

7) $x = 1-t$ $\quad 0 \le t \le 1$
$y = t$

$$\int_C xy\,dx + y^2\,dy = \int_0^1 ((1-t)t\cdot(-1) + t^2(1))\,dt$$

$$= \int_0^1 t^2 - t + t^2\,dt = \frac{2}{3}t^3 - \frac{t^2}{2} = \boxed{\frac{1}{6}}$$

8) a)



$\vec{r}_1(t)$: $x(t) = t$ $\quad -1 \le t \le 2$
$y(t) = t^2$

$\vec{r}_2(t)$: $x(t) = 2 + at = 2 - 3t$ $\quad 0 \le t \le 1$
$y(t) = 4 + at = 4 - 3t$

$$\Rightarrow \int_{C_1} M\,dy - N\,dx + \int_{C_2} M\,dy - N\,dx = \int_{-1}^{2} (2t)(2t) - (t - t^2)\,dt$$
$$+ \int_0^1 (2-3t)(2)(-3) - (2-3t-4+3t)(-3)\,dt$$
$$= \boxed{9/2}$$

b) the rate at which a "fluid" is entering or leaving a region.

E.C.

$0 \le \rho \le \sec\varphi$
$0 \le \varphi \le \arctan(\csc(\theta))$
$\pi/4 \le \theta \le \pi/2.$