Midterm II Soln.

1) a) product rule

$$(3x^2+4x)\left(5x^6-7x^3+\frac{8}{11}\right)+(x^3+2x^2)(30x^5-21x^2)$$

b) product rule

$$100x^{99}\sin(2x)+x^{100}\cos(2x)\cdot 2$$

c) quotient rule

$$\frac{(4x^2-3x+1)-(x-2)(8x-3)}{(4x^2-3x+1)^2}$$

d) chain rule

$$3\left(\tan\left(\tfrac{x}{2}\right)\right)^2\left(\sec^2\left(\tfrac{x}{2}\right)\right)\cdot\frac{1}{2}.$$

2) $f'(x)=\frac{1}{5}(x+2)^{-4/5}$ $f''(x)=\frac{1}{5}\cdot\frac{-4}{5}(x+2)^{-9/5}$ $f'''(x)=\frac{1}{5}\cdot\frac{-4}{5}\cdot\frac{-9}{5}(x+2)^{-14/5}$.

$$\Rightarrow \boxed{f^{(3)}(0)=\frac{4\cdot 9}{5^3}(2)^{-14/5}.}$$

3) $$\frac{f(4)-f(1)}{4-1}=\frac{\left(\frac{2(4)^2+3}{2\sqrt{4}+1}\right)-\left(\frac{2(1)^2+3}{2\sqrt{1}+1}\right)}{3}=\boxed{\frac{16}{9}}$$

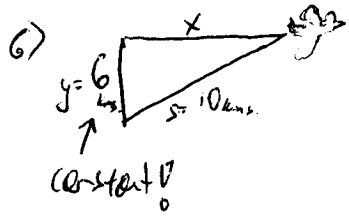
4) a) $h=-16t^2+64t+80$, water $\Rightarrow h=0 \Rightarrow 0=-16t^2+64t+80$

$$=-16(t^2-4t-5)=-16(t-5)(t+1)$$

$\Rightarrow t=-1, (5)$.

b) velocity $\Rightarrow h' \Rightarrow v=h'=-32t+64$ so at $t=5$, $v(5)=-32\cdot 5+64=\boxed{-96}$

5) A continuous function on a closed interval must achieve an absolute max and an absolute min.

6) 

$x^2+y^2=s^2 \Rightarrow x^2=s^2-y^2=100-36=64 \Rightarrow x=8.$

Speed of bird - how fast it's going horizontally, i.e. $\frac{dx}{dt}$

take derivative, $2x\frac{dx}{dt}+2y\frac{dy}{dt}=2s\frac{ds}{dt}$

↑ y is constant, $\frac{dy}{dt}=0$

$$\Rightarrow 2x\frac{dx}{dt}=2s\frac{ds}{dt}$$

$$\frac{dx}{dt}=\frac{s}{x}\frac{ds}{dt}=\frac{10}{8}\cdot 240=\boxed{300\ \text{km/hr}}\ !$$

7) a) $f'(x)=1-\sin(x)$ c.p. $f'(x)=0 \Rightarrow 0=1-\sin(x)$, $\sin(x)=1 \Rightarrow \boxed{x=\pi/2}$.

b) $$f'(x)=\frac{(x^2-12)3x^2-x^3(2x)}{(x^2-12)^2}=\frac{3x^4-36x^2-2x^4}{(x^2-12)^2}=\frac{x^4-36x^2}{(x^2-12)^2}=\frac{x^2(x^2-36)}{(x^2-12)^2}$$

$f'(x)=0$ when $x=0$ or ± 6. Undefined when $x=\pm\sqrt{12}$.

8) a) $f'(x) = 3x^2 - 12 = 3(x^2-4)$ so $f'(x)=0$ when $x = \pm 2$.

$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$x = -3$	$x = 0$	$x = 3$
$f'(-3) > 0$	$f'(0) < 0$	$f'(3) > 0$
↑	↓	↑

b) On $[-3,3]$ - plug into f.

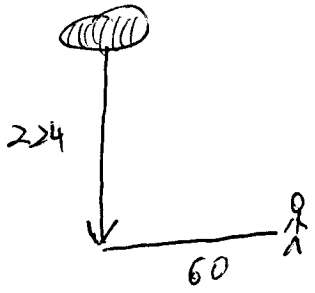
$\Rightarrow$ $f(-3) = -27 + 36 + \pi/8 = 9 + \pi/8$

$f(-2) = -8 + 24 + \pi/8 = (+16 + \pi/8)$ - largest - abs. max

$f(2) = 8 - 24 + \pi/8 = (-16 + \pi/8)$ - smallest - abs. min.

$f(3) = 27 - 36 + \frac{\pi}{8} = -9 + \pi/8$

E.C.



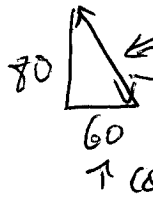
$h = -16t^2 + 224$. ($V_0 = 0$!)

$V = -32t$.

so after 3 sec,

$h = 80$, $V = -96$.

$\Rightarrow$



← must be 100 ($60^2 + 80^2 = s^2$).

distance between you and watermelon

↑ constant

$\Rightarrow$ $x^2 + y^2 = s^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2s\frac{ds}{dt}$

$x\frac{dx}{dt} = 0$

$\frac{ds}{dt} = \frac{y}{s}\frac{dy}{dt} = \frac{80}{100} \cdot -96 = \boxed{\frac{-384}{5}\ \text{ft/sec}}$