1) a) $f(x,y) = \sqrt{4 - x^2 - 2y^2 + x}$

$\sqrt{\text{must be bigger than or equal to } 0}$!

$$\Rightarrow \quad 4 - x^2 - 2y^2 + x \geq 0$$

$$\Rightarrow \quad 4 \geq x^2 - x + 2y^2 \cancel{\geq 0}$$

complete the square!

$$\Rightarrow \quad 4 + \frac{1}{4} \geq \left(x - \frac{1}{2}\right)^2 + 2y^2 \qquad x^2 - x + \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \frac{17}{4} \geq \left(x - \frac{1}{2}\right)^2 + 2y^2$$

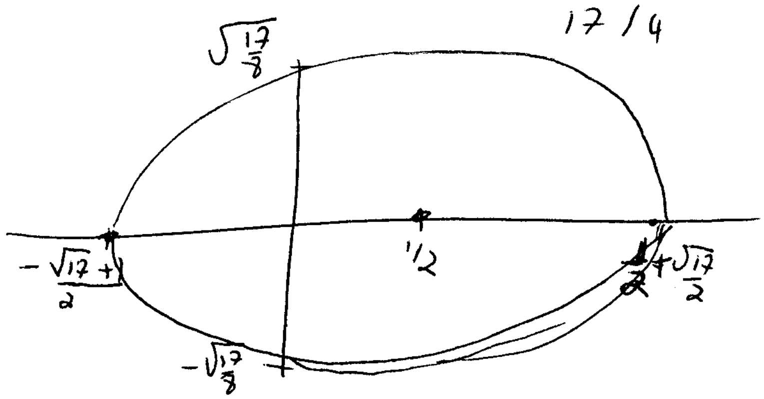
— ellipse! & everything inside it.

$$1 \geq \frac{\left(x - \frac{1}{2}\right)^2}{17/4} + \frac{2y^2}{17/4}$$

centered at $\left(\frac{1}{2}, 0\right)$

axis in x direction $\sqrt{\frac{17}{4}}$ each way

axis in y direction $\sqrt{\frac{17}{8}}$



So the domain is everything on this ellipse and inside it — that is $4 \geq x^2 - x + 2y^2$.

D: $4 \geq x^2 - x + 2y^2$

The range has its smallest point when x & y are on the ellipse — that is $4 = x^2 - x + 2y^2$

Range: $\left[0, \sqrt{\frac{17}{4}}\right]$

$$\Rightarrow \quad \sqrt{4 - (x^2 - x + 2y^2)} = \sqrt{4-4} = 0.$$

and it's largest point when x and y are the furthest from the ellipse — so at the center $\Rightarrow \sqrt{4 - \left(\frac{1}{2}^2 - \frac{1}{2} - 0\right)} = \sqrt{\frac{17}{4}}$.