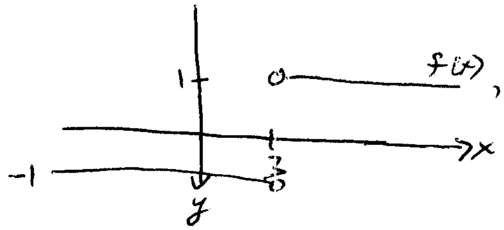
One-Sided Limits

A one-sided limit is defined as the limit taken from one side.

Hence for



$$f(x) = \frac{|x-3|}{x-3}$$

$\lim_{x \to 3^+} \frac{|x-3|}{x-3} = 1$ since as you get closer and closer to 3 from the right, you are at 1 consistently.

meanwhile, $\lim_{x \to 3^-} \frac{|x-3|}{x-3} = -1$ since this time as we approach 3 from the left we are consistently at -1.

Note that these limits exist even though at 3, $f(x)$ is not defined!

If we defined a new function $g(x) = \begin{cases} \frac{|x-3|}{x-3}, & \text{if } x \neq 3 \\ 1, & \text{if } x = 3 \end{cases}$

then now for g, $\lim_{x \to 3^+} \frac{|x-3|}{x-3} = 1$,

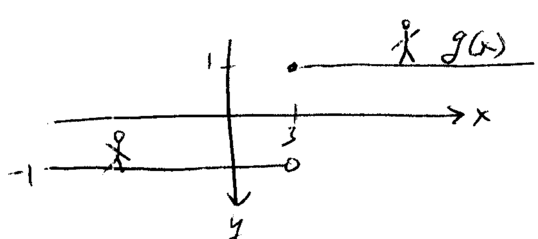
$\lim_{x \to 3^-} \frac{|x-3|}{x-3} = -1$,

and g is defined at 3! But because the limit from the left and the limit from the right are not equal,

$$\lim_{x \to 3^-} \frac{|x-3|}{x-3} \neq \lim_{x \to 3^+} \frac{|x-3|}{x-3}$$

then $\lim_{x \to 3} \frac{|x-3|}{x-3}$ is undefined. This is because we were not told which way to approach 3 (from the left? from the right?) and hence don't know which answer to take.

Using stick figures,



they would never meet – hence no limit!!!