Pascal's triangle:

$$
\begin{array}{ccccccccccccccl}
 & & & & & & 1 & & & & & & & \text{row } 0\\
 & & & & & 1 & & 1 & & & & & & \text{row } 1\\
 & & & & 1 & & 2 & & 1 & & & & & \text{row } 2\\
 & & & 1 & & 3 & & 3 & & 1 & & & & \text{row } 3\\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & & \text{row } 4\\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & \text{row } 5\\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \text{row } 6\\
 & & & & & & \vdots & & & & & & &
\end{array}
$$

Rules: 1's on outside.

Any number inside is gotten by adding the two numbers above it, i.e.

$$
\begin{array}{ccc}
5 & & 10\\
 & 15 & \\
\end{array}
\quad \ldots
$$

How to use Pascal's triangle:

when foiling, $(x+y)^n$, n a whole number, Pascal's Δ can be used to find the coefficients.

Ex. Find $(x+y)^6$

- First write the powers of x in descending order (highest power is 6)

$$x^6 \quad x^5 \quad x^4 \quad x^3 \quad x^2 \quad x^1 \quad x^0$$

- Add the powers of y in ascending order,

$$x^6y^0 \quad x^5y^1 \quad x^4y^2 \quad x^3y^3 \quad x^2y^4 \quad x^1y^5 \quad x^0y^6$$

- Since we are taking the 6th power, go to the 6th row - those are the coefficients in that order!

$$1x^6y^0 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6x^1y^5 + 1x^0y^6$$

$$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6.$$

Ex. Find $(x^2+3)^4$

- $(x^2)^4(3)^0 \quad (x^2)^3(3)^1 \quad (x^2)^2(3)^2 \quad (x^2)^1(3)^3 \quad (x^2)^0(3)^4$

- row 4 (4th power) $\Rightarrow (x^2)^4 + 4(x^2)^3(3)^1 + 6(x^2)^2(3)^2 + 4(x^2)^1(3)^3 + (3)^4$

$$= x^8 + 12x^6 + 54x^4 + 108x^2 + 81. \text{ (after multiplying everything out).}$$