Midterm review II

1) a) $f'(x) = (15x^4)(x^4 - 18x^2 + 24) + (3x^5 - 17)(4x^3 - 36x)$

b) $(\cos(x))' = -\sin(x)$, $(\sin(x))' = \cos(x)$, $(\tan(x))' = \sec(x)\tan(x)$, $(\csc(x))' = -\csc(x)\cot(x)$

c) $f'(x) = \dfrac{(3x^2+1)(-\sin(x)) - (\cos(x)+4)(16x)}{(3x^2+1)^2}$

second c...

d) $f''(x) = (3x + 2x^{-1})^{1/5}$ $\quad f'(x) = \frac{1}{5}(3x+2x^{-1})^{-4/5} \cdot (3 - 2x^{-2})$.

d) $\dfrac{(5x+1)(2(4x^5 - 3x + 7)(20x^4 - 3) - (4x^5 - 3x + 7)^2(5)}{(5x+1)^2}$

e) $\left(\cot(3x+1)\right)' = -\csc^2(3x+1)(3)$.

2) a) $f'(x) = -\sin(x)$ $\quad f''(x) = -\cos(x)$ $\quad f'''(x) = +\sin(x) \Rightarrow f'''(0) = 0$.

b) $f(x) = (x+1)^{11/3} \Rightarrow f'(x) = \frac{1}{3}(x+1)^{-2/3}$ $\quad f''(x) = \frac{1}{3}\cdot\frac{-2}{3}(x+1)^{-5/3}$

$f'''(x) = \frac{+1}{3}\cdot\frac{-2}{3}\cdot\frac{-5}{3}(x+1)^{-8/3}$ at $0 \Rightarrow f'''(0) = \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{5}{3} = \frac{10}{27}$.

3) a) $2y\dfrac{dy}{dx} + 3x^2 = 8$

b) $3y^2 + 3x\cdot 2y\cdot\dfrac{dy}{dx} + 15 = \dfrac{-\sin(x)}{3}$

c) $3(2x^2y^4 - 5)^2 \cdot \left(4xy^4 + 2x^2\cdot 4y^3\dfrac{dy}{dx}\right) = 0$.

4) $\Rightarrow$ on $[0,2]$, note that $\frac{2x+7}{x^2}$ is not defined on 0!

$\Rightarrow$ average rate of change $\dfrac{f(2) - f(0)}{2-0}$ is undefined!

on $[-5,-1]$ $\quad \dfrac{f(-5) - f(-1)}{-5 - -1} = \dfrac{\frac{(-10+7)}{25} - \frac{(-2+7)}{1}}{-4}$

$= \dfrac{\frac{-17}{25} + 5}{-4} = \dfrac{\frac{-17}{25} + \frac{25}{25}}{-4} = \dfrac{\frac{8}{25}}{-4} = \dfrac{8}{-100} = \boxed{\dfrac{-2}{25}}$

5) $h = -16t^2 + 48t + 64$.

hits water $\Rightarrow h = 0$ $\quad 0 = -16(t^2 - 3t - 4)$

$= -16(t-4)(t+1) \Rightarrow t = 4, -1$

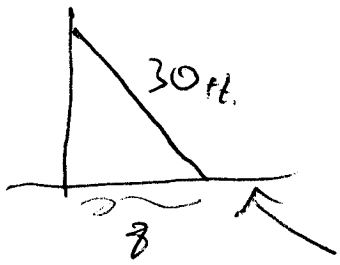
(make sense; -1 sec?)

velocity $\Rightarrow \frac{dh}{dt} = -32t + 48$

$\Rightarrow -32(4) + 48 = \boxed{-80}$

$a = \frac{d^2h}{dt^2} = h'' = \boxed{-32}$

6)



$l = 30$ ft $\quad \frac{dx}{dt} = 4$ ft/sec.

$x = 8$ ft.

triangle $\Rightarrow x^2 + y^2 = 30^2$.

solve for $y \Rightarrow y = \sqrt{900 - 64}$

take derivative $\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$.

plug in $\Rightarrow \frac{dy}{dt} = \frac{-2x\frac{dx}{dt}}{2y} = \frac{-2(8)\cdot 4}{2\cdot\sqrt{900-64}}$

7) a) $f'(x) = \frac{2x}{4} - 4$. $\quad \frac{x}{2} - 4 = 0$ when $x = 8$.

$(-\infty, 8) \quad (8, \infty)$

$x = 1 \quad x = 10$

$f'(1) < 0 \quad f'(10) >$

$\downarrow$ dec. $\quad \uparrow$ inc.

$\left((-1)^2 - 9\right)^{-1/5} = \frac{1}{(-8)^{1/5}}$

fifth root of neg. # is negative.

b) $f'(x) = \frac{4}{5}(x^2-9)^{-1/5} \cdot 2x = 0$ when $x = 0$

undefined at $x = \pm 3$.

$\Rightarrow$ a $[-4, 5]$

$[-4, -3) \quad (-3, 0) \quad (0, 3) \quad (3, 5]$

$x = -3.5 \quad x = -1 \quad x = 1 \quad x = 4$

$f'(4) < 0 \quad f'(-1) > 0 \quad f'(1) < 0 \quad f'(4) > 0$

$\downarrow \quad \uparrow \quad \downarrow \quad \uparrow$

7) c) $f(x) = 2\cos(x)$ on $[0, 2\pi)$

$f'(x) = -2\sin(x)$. $f'(x) = 0$ when $x = 0, \pi$ (2π not in the interval!)

$\Rightarrow$ $[0, \pi)$ $(\pi, 2\pi)$

$x = \pi/2$ $x = 3\pi/2$

$f'(\frac{\pi}{2}) = -2 < 0$ $f'(\frac{3\pi}{2}) = 2 > 0$

$\downarrow$ $\uparrow$

8) for a) relative min will be at $x=8$,

$\downarrow$ before, $\uparrow$ after.

for b) must check endpoints of intervals!!

1st, $x=-3$ min, $x=0$ max, $x=3$ min.

$f(-4) = (7)^{4/5}$ $f(0) = (-9)^{4/5}$ $f(5) = (16)^{4/5}$

$f(-3) = (0)^{4/5}$ $f(3) = (0)^{4/5}$

$\uparrow$ max.

mins.

$((-9)^{1/5})^4 > 0$!!

c) can see that at π we get a min.

need to check at endpoints,

$F(0) = 2$, at $F(2\pi) = 2$.

$\Rightarrow$ absolute max