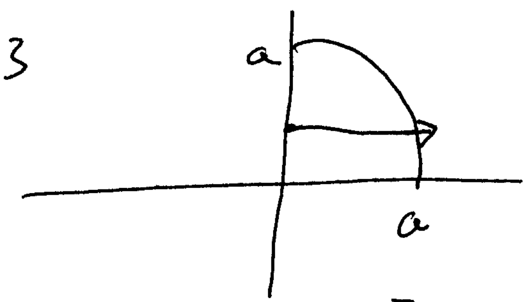
$$\Rightarrow \int_0^1 \int_0^{4x^2} e^{x^3}\,dy\,dx = \int_0^1 4x^2 e^{x^3}\,dx$$

$$u = x^3 \qquad du = 3x^2\,dx$$

$$\Rightarrow \frac{4}{3}\int_0^1 e^u\,du \Rightarrow \frac{4}{3}\, e^{x^3}\Big|_0^1 = \boxed{\frac{4}{3}(e-1)}$$

2) $\delta(x,y) = x^2 y + 3$



$\Rightarrow \quad \bar{x} = \dfrac{M_y}{M}$ | $\bar{y} = \dfrac{M_x}{M}$

$$M_y = \iint x(x^2y+3)\,dx\,dy$$

$$M_x = \iint y(x^2y+3)\,dx\,dy = \int_0^a \int_0^{\sqrt{a^2-y^2}} y(x^2y+3)\,dx\,dy.$$

(note that order doesn't matter due to the symmetry of the region here)

$$= \int_0^a \int_0^{\sqrt{a^2-y^2}} x(x^2y+3)\,dx\,dy$$

w/ $$M = \int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2y+3)\,dx\,dy.$$

3)

$$M = \int_0^1 \int_0^6 (x+y+1)\, dx\, dy = 27$$

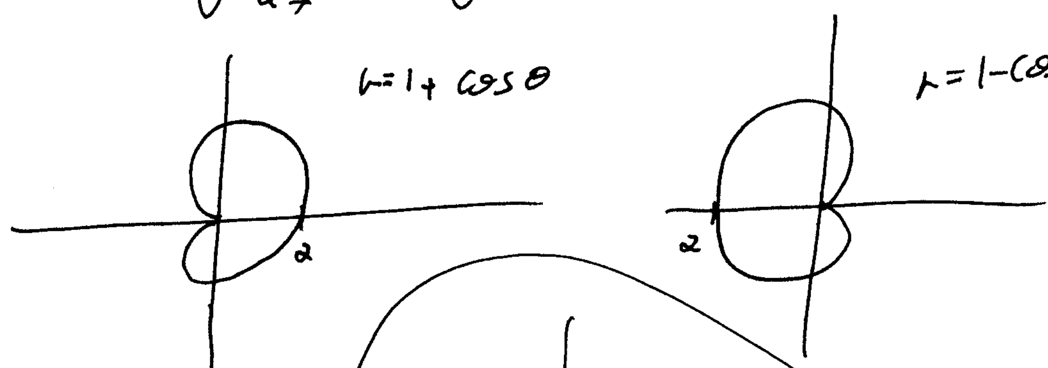
$$M_y = \int_0^1 \int_0^6 x\,(x+y+1) = 99 \qquad M_x = \int_0^1 \int_0^6 y\,(x+y+1) = 14.$$

$$\Rightarrow \bar{x} = \frac{99}{27} = \boxed{\frac{11}{3}} \qquad \bar{y} = \boxed{\frac{14}{27}}$$

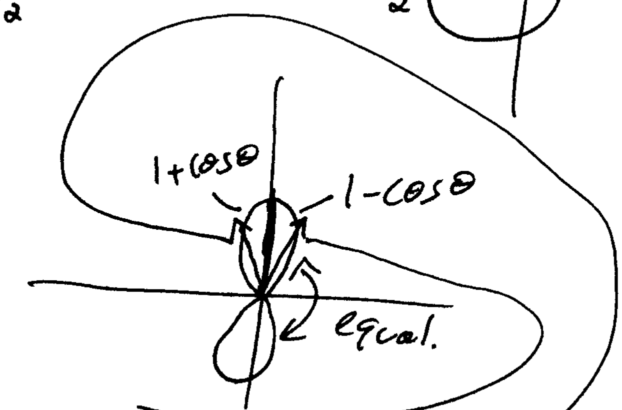
$$I_y = \int_0^1 \int_0^6 x^2(x+y+1)\, dx\, dy = \boxed{432.}$$

$$R_y = \sqrt{I_y/M} = \sqrt{\frac{432}{27}} = \sqrt{16} = \boxed{4.}$$

4) Sketch!



$\Rightarrow$ what



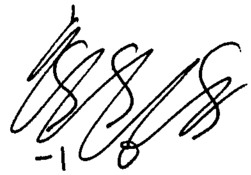
$$\Rightarrow 2\cdot\left(\int_0^{\pi/2}\int_0^{1-\cos\theta} r\, dr\, d\theta + \int_{\pi/2}^{\pi}\int_0^{1+\cos\theta} r\, dr\, d\theta\right)$$

You have to divide this in half! since r goes from the origin out!

Note that in fact each half is equal!

$$\Rightarrow = 4\int_0^{\pi/2}\int_0^{1-\cos\theta} r\, dr\, d\theta = \boxed{\frac{3\pi}{2} - 4}$$

(Only tricky part is $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$).

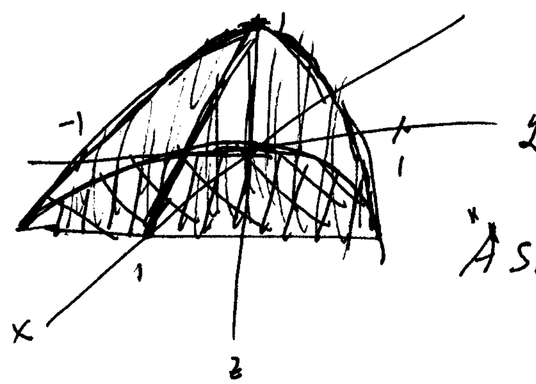
⑤ 

$\cancel{x}$
$\cancel{y}$
z

$0 \le z \le 1-x$

$y^2 \le x \le 1$

$-1 \le y \le 1$



"A slice of an orange".

$\Rightarrow$ $0 \le y \le 1$

$0 \le z \le x^2$

$-1 \le x \le 0$

$y^2 \le \cancel{x} \le 1-\cancel{z}$

$0 \le z \le 1-y^2$

$-1 \le y \le 1$

$y^2 \le x \le 1-z$

$-\sqrt{1-z} \le y \le \sqrt{1-z}$

$0 \le z \le 1$

$-\sqrt{x} \le y \le \sqrt{x}$

$0 \le x \le 1-z$

$0 \le z \le 1$

$0 \le z \le 1-x$

$-\sqrt{x} \le y \le \sqrt{x}$

$0 \le x \le 1.$

6) (43) $\displaystyle\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3} \frac{\pi e^{2x}\sin\pi y^2}{y^2}\,dx\,dy\,dz = \int_0^1\int_{\sqrt[3]{z}}^1 \frac{\pi\sin\pi y^2}{y^2}\,\frac{e^{2x}}{2}\Big|_0^{\ln 3} dy\,dz$

$\displaystyle = \int_0^1\int_{\sqrt[3]{z}}^1 \frac{\pi\sin\pi y^2}{y^2}\left(\frac{e^{2\ln 3}-1}{2}\right)dy\,dz = \int_0^1\int_{\sqrt[3]{z}}^1 \frac{4\pi\sin\pi y^2}{y^2}\,dy\,dz.$

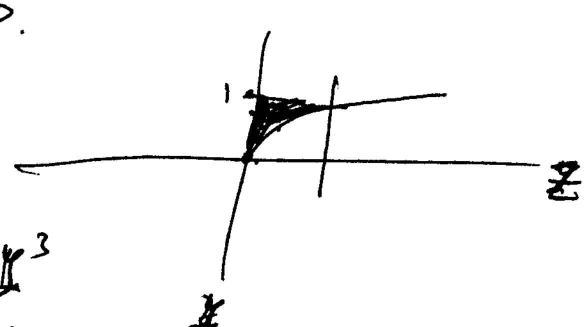
$\Rightarrow \sqrt[3]{z} \le y \le 1$ ~~right~~/top.

left/bottom $0 \le z \le 1$

↑ depends on how you draw your axis!

$\Rightarrow$ $0 \le z \le y^3$

$0 \le y \le 1$



6) cont. $\Rightarrow \displaystyle\int_0^1 \int_0^{y^3} \frac{4\pi \sin \pi y^2}{y^2}\, dz\, dy$

$$= \int_0^1 \frac{4y^3}{y^2}\, \pi \sin \pi y^2\, dy$$

$u = \pi y^2$

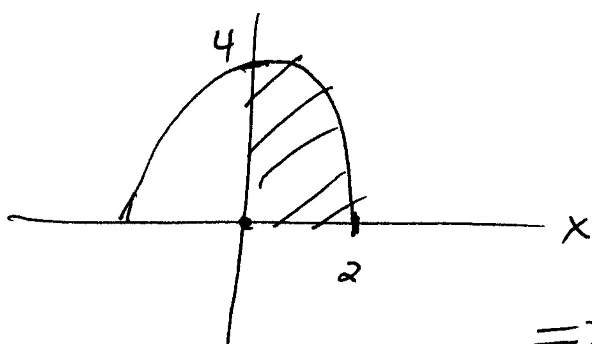
$du = 2\pi y\, dy.$

$$\Rightarrow \frac{4}{2}\int_0^{\pi} \sin u\, du = -2\cos(\pi y^2)\Big|_0^1 = -2(-1) + 2(1) = \boxed{4}$$

44) $\displaystyle\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z}\, dy\, dz\, dx.$

$$= \int_0^2 \int_0^{4-x^2} \frac{x \sin 2z}{4-z}\, dz\, dx$$

more abstract cut.



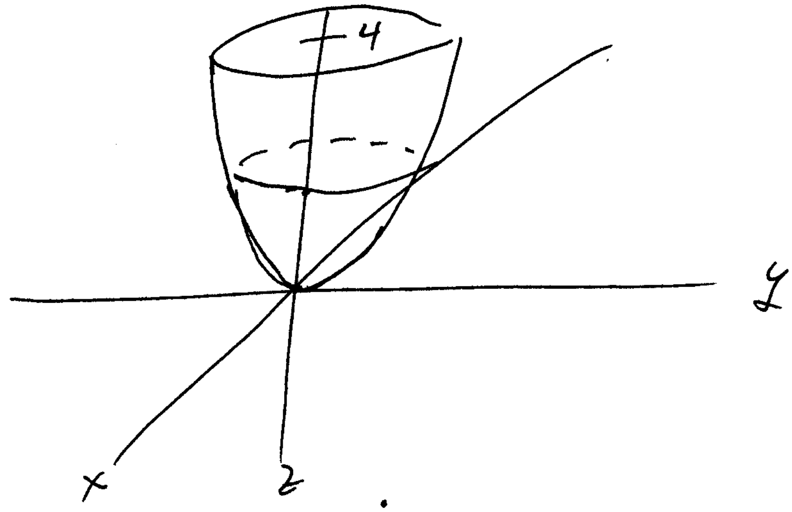
$\Rightarrow$ $0 \le x \le \sqrt{4-z}$

$0 \le z \le 4.$

$$= \int_0^4 \int_0^{\sqrt{4-z}} \frac{x \sin 2z}{4-z}\, dx\, dz = \frac{1}{2}\int_0^4 \cancel{\frac{(\sqrt{4-z})^2}{4-z}}\, \sin 2z\, dz$$

$$= \frac{1}{2}\int_0^4 \sin 2z\, dz = -\frac{1}{4}\cos 2z\Big|_0^4 = \boxed{\frac{1}{4} - \frac{1}{4}\cos 8}$$

7)



$\bar{x}, \bar{y}$ will be 0.
due to Symmetry!

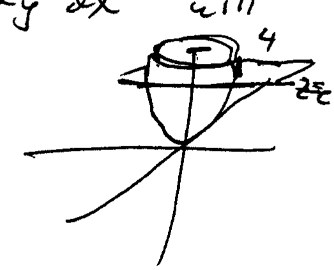
$$M = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} 1 \cdot dz\,dy\,dx = \boxed{8\pi} \quad \text{(use polar!)}$$

$$M_{xy} = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} z\,dz\,dy\,dx = \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 8 - (x^2+y^2)^2\,dy\,dx$$

$$= \int_{0}^{2\pi}\int_{0}^{2} \left(8 - \frac{r^4}{2}\right) r\,dr\,d\theta = \frac{64\pi}{3} \Rightarrow \boxed{\bar{z} = \frac{8}{3}}$$

So center of mass is $\left(0, 0, \frac{8}{3}\right)$.

To find C, note that $\int_{-\sqrt{c}}^{\sqrt{c}}\int_{-\sqrt{c-x^2}}^{\sqrt{c-x^2}}\int_{x^2+y^2}^{c} dz\,dy\,dx$ will give us half the mass $\Rightarrow = 4\pi$!



$$\Rightarrow \int_{-\sqrt{c}}^{\sqrt{c}}\int_{-\sqrt{c-x^2}}^{\sqrt{c-x^2}} c - (x^2+y^2)\,dy\,dx \Rightarrow \text{polar}$$

$$\Rightarrow \int_{0}^{2\pi}\int_{0}^{\sqrt{c}} (c - r^2)\,r\,dr\,d\theta = \int_{0}^{2\pi} \frac{c^2}{4}\,d\theta = \frac{c^2\pi}{2}.$$

$$\Rightarrow \frac{c^2\pi}{2} = 4\pi \Rightarrow c^2 = 8 \Rightarrow \boxed{c = 2\sqrt{2}}$$