

## Sample Exam for 16C Hillel Raz

Note that this is a sample exam and does not reflect the length of the actual test. The actual midterm will be shorter, though the types of questions will be similar and based on the material covered here.

### Differential Equations (Appendix C)

- Know how to verify that an equation is a solution of a differential equation.
- Know how to find a particular solution.
- Know when you can use separation of variables to get to a general solution (separable vs. inseparable).
- Know when an equation is a first-order linear equation and how to solve it (integrating factor method).
- Know how to write a model given a word problem and how to solve it, including finding the particular solution.

1. Verify that  $3y^2 + x^2y = C$  is the general solution of the differential equation  $2xy + y'(x^2 + 6y) = 0$ . Find the particular solution given  $y = -1$  when  $x = 2$ .

2. Solve the following equations using whatever method suitable.

a.  $\frac{dy}{dx} = \frac{e^x}{4y}$

b.  $xy' - 4xe^{x^4} = -3y; \quad x > 0$

c.  $y' + y = 6e^x$

d.  $y' = \frac{x}{y} - \frac{x}{1+y}$

2. A 100 gallon tank is half full of distilled water. At time  $t = 0$ , a solution containing 1.5 pound of concentrate per gallon enters the tank at the rate of 10 gallons per minute, and the well-stirred mixture is withdrawn at the same rate. Find the amount  $Q$  of concentrate after 30 minutes.

3. The rate of change of a population of monkeys is proportional to the square root of the monkey population. Initially there are 100 monkeys and 2 years later there are 400 monkeys. How many monkeys will there be in 5 years?

4. Also know how to do a problem like we did in class involving the Gompertz growth model (numbers 23, 24 on p. A45). You will be given the model itself so no need to memorize that.

## Chapter 7

- Know the three dimensional coordinate system, the distance and the midpoint formulas.
- Know the equation for a sphere and how to get it using the distance formula, the midpoint formula or by completing the square.
- Know the equation for a plane and how to draw a plane given an equation.
- Know how to identify the different quadric surfaces, how to sketch and describe the traces and based on those, how to draw the surface itself.

5. Find the sphere's center and radius starting with the following equation:

$$4x^2 + 4y^2 + 4z^2 - 8x + 12z + 7 = 0$$

6. Find the standard form of the equation of a sphere for a sphere with the endpoints of a diameter on  $(2, 4, 5)$  and  $(7, -3, 0)$ .

7. Find the intercepts and sketch the graph of a plane satisfying the following equation:

$$2x - y + 5z = 8.$$

8. Sketch and identify the traces for the following quadric surfaces, then identify the surfaces and sketch them.

a.  $z^2 = 9x^2 + y^2$ .

b.  $-2x^2 + y^2 + 5z^2 = 1$ .

c.  $2y = x^2 - z^2$ .