Sample Exam 1 for 21D Hillel Raz

Note that this is a study guide and does not reflect the length of the actual test. The concepts covered here are the concepts that will be covered on the exam (not necessarily all of the ones covered here will be on the exam though). The questions will be similar for the most part. Make sure to review examples done in class and in the book. The questions on the exam won't be much different than homework questions.

This exam will cover material from chapter 15, sections 15.1-15.6. You will need to know the following general concepts:

Sections 15.1 and 15.2:

- Double integrals - what are they, what do they mean?
- How to evaluate them.
- How to to switch the order of integration.
- How to find the area of a region using double integrals.
- Average value of a function, first moments and center of mass, second moments and radii of gyration.

1. Double integrals such as:
$\int_0^1 \int_{1-x}^{1+x} \frac{2y}{x+1} dydx$
$\int_0^4 \int_{\sqrt{y/4}}^1 e^{x^3} dxdy$

p. 1063 problems 10, 32, 34, 35, 51 are also good (you could also look at questions from the chapter review and try to do anyone that seems difficult.)

2. Set up but do not evaluate the integrals for finding the center of mass for a solid of density $\delta(x, y) = x^2y + 3$ over the region cut from the first quadrant by the cirlce $x^2 + y^2 = a^2$.

3. Find the moment of inertia and radius of gyration about the y-axis of a thin rectangular plate cut from the first quadrant by the lines x = 6 and y = 1 if $\delta(x, y) = x + y + 1$.

There are many similar problems in the book, section 15.2 and/or the chapter review.

Sections 15.3-15.5:

- Know the meaning of polar coordinates and how the correct expression for dA in polar coordinates.
- Be able to write the limits of integrations in polar coordinates (hence sketch the region given by a polar equation) and integrate accordingly.
- Know how to calculate the first moment and second moments of 2-dimensional objects in polar coordinates.
- Be able to set up triple integrals in all possible (6) orders.
- Average value and volume for triple integrals.
- Know how to calculate masses and moments in three dimensions (no need to worry about cylindrical or spherical coordinates on this exam).

4. Find the area of the region common to the interiors of the cardioids $r = 1 + cos\theta$ and $r = 1 - cos\theta$. (and similar problems - 11, 24, 29 in section 15.3 are pretty good)

5. For the following triple integral, rewrite the equivalent triple integrals in all possible orders:

$\int_{-1}^{1}\int_{y^2}^{1}\int_{0}^{1-x} dzdxdy$. Then evaluate one of the triple integrals. (problems 21 - 30 on 15.4 are similar and all good representatives)

6. Problems 43, 44 and that type on 15.4.
7. Find the center of mass of a solid of constant density bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane z = 4. Find the plane z = c that cuts this solid into two parts of equal mass. (hint - draw the plane and set up the integrals). (other similar problems are in 9, 10 and others in section 15.5)