Sample Exam 2 for 16C Hillel Raz

Note that this is a sample exam and does not reflect the length of the actual test. The actual midterm will be shorter, though the types of questions will be similar and based on the material covered here. The questions on the exam might be harder or easier, the difficulty here is not necessarily representative.

Functions of Several Variables (Chapter 7, sections 7.3-7.6, 7.8-7.9)

- Know how to find the domains and ranges of functions of several variables, and how to compute them.

1. Describe and (know how to sketch as well!) the domain and the range of the following functions:
a) f(x, y) = $\sqrt{4 - x^2 - 2y^2 + x}$

b) f(x, y, z) = $\dfrac{z^2}{x + y}$

c) f(x, y) = ln(1 - $\dfrac{1 - x}{1 - y}$)

- Know how to compute (first and second) partial derivatives of functions of multiple variables.

2. (only one question 2 this time...) Find $f_x,\ f_{xx},\ f_{xy},\ f_y$ and f_{yy} for the following:
a) $f(x, y) = xy^3 + (x - y)^2 + 2$

b) $f(x, y) = xe^{xy-y-1}$

c) $f(x, y) = ln(x - \dfrac{12}{y})$

- Know how to find extrema point of functions of multiple variables and how to identify them.
- Know the exact formula for computing the second partials test for relative extrema.

3. Find the extrema points of the following equations and identify them as maximum, minimum or saddle points.
a)$f(x, y) = x^2 + 2xy + y^2$

b)$f(x, y) = x + y^2 - e^x$

c)$f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 8$ (there are four critical points here...)

4. Use the second partials test to find three positive numbers, x, y and z, such that their sum is 8 and their product is a maximum (note that this could be done also with Lagrange multipliers - what would the product correspond to?)

- Know how to find the maximum and minimum of functions of multiple variables under specific constraints.
- Know the Lagrange method for optimized constraint (or constrained optimization, depends on which verb modifies which noun...)

5. Numbers 35 and 49 on p. 513.

- Know how to take integrals with respect to only one variable.
- Be very familiar with double integrals and the order of integration.
- Know how to find the area between two curves.
- Be able to write the double integral of an area in two ways (i.e., first integrating wrt x and then wrt y, or vice versa).

6. Let R be the region in xy plane bounded by the functions y = lnx, x = 1 and y = 3. Write the integral for the area in two ways and solve one of the double integrals you have written for the area of the region.

7. p. 531, numbers 38 and 39.

- Know how to calculate the volume of a solid using double integrals.
- Know how to calculate the average value of a function over a region.

8. p. 539, numbers 15-26 are good representatives!

9. Find the average value of f(x, y) = $\frac{x}{y}$ over the region y = lnx, x = 1, and y = 3.