Sample Exam 2 for 21D Hillel Raz

Note that this is a study guide and does not reflect the length of the actual test. The concepts covered here are the concepts that will be covered on the exam (not necessarily all of the ones covered here will be on the exam though). The questions will be similar for the most part. Make sure to review examples done in class and in the book. The questions on the exam won't be much different than homework questions.

This sample exam provides sample questions which should up to a bit more than the length of the exam (aobut one hour and twenty minutes for this sample while the exam is an hour), hence you can time yourself if you'd like to practice in a timed envirnoment...

This exam will cover material from chapter 15, sections 15.6,15.7 and from chapter 16, sections 16.2, 16.2. You will need to know the following general concepts:

Sections 15.6 and 15.7:

- Cylindrical coordinates - what are they, what order is it good to integrate them in, what is their relationship to the cartesian coordinates (x, y, z).
- How to switch the order of integration using cylindrical coordinates.
- Spherical coordinates - what are they, what order is it good to integrate them in, what is their relationship to the cartesian coordinates (x, y, z).
- How to to switch the order of integration using spherical coordinates.
- Know how to evaluate integrals in terms of cylindrical and spherical coordinates.
- Know what the dV term is for both cylindrical and spherical coordinates.

1. Set up and evaluate the integrals for the volume of the following solids:
a) A right circular cylinder whose base is the circle r = 3 cos θ and whose top lies in the plane z = 5 - x.
b) The solid enclosed by the cardioid of revolution ρ = 1 - cosϕ.

There will be questions similar to numbers 31 and 32 on p. 1109-1110 which we did in class so try doing those without any from your notes. Also problems like those in the homework - look at section 15.6, there are lots of problems in there. It is possible I will take one of those and put them on the exam, possibly in an altered form...

Section 15.7:
- Know how the Jacobian is defined for both 2-D and 3-D and of course, know how to compute it...
- Given a transformation, know how to rewrite the integrals in terms of the new variables and be able to evaluate them.
- Know how to dervie the term r dz dr dθ using the Jacobian and cylindrical coordinates.
- Know how to derive the term $\rho^2 \, sin\phi \, d\rho \, d\phi \, d\theta$ using the Jacobian and spherical coordinates.

2. Use the transformation x = u + (1/2)v, y = v to evaluate the following integral (or if you'd like, you can try evaluating it straight up...):
$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x-y)e^{(2x-y)^2} dxdy$$

Questions like number 20, 21 and 24 in the section are also very nice.

Section 16.1:
- Know how to parametrize curves in space.
- Know what a line integral is, why its different than a 'normal' integral and how to evaluate it.
- Know how to use line integrals to find the different moments of coil springs, thing rods, wires and other items that can be approximated using a line integral.

3. Integrate $f(x, y) = x^3/y$ over the curve given by $C_1 : y = x^2/2, 0 \leq x \leq 2$ and C_2 : the line segment between (2, 2) and (4, 3).

4. Find I_x for the wire given by the semicircle $y^2 + z^2 = 1, z \geq 1$, in the yz plane, if the density of the wire is given by $\delta(x, y, z) = 2 - z$.

Other problems as usual, similar to what we did in class and/or in the homework... (you can look in the section, or in the chapter review if you want more practice)

Section 16.2:
- Know the following concepts and be able to explain what their significance is: the gradient and the flux.
- Know how to set up and evaluate a work integral over a curve.
- Know how to set up and evaluate a flow integral over a curve.
- Know how to set up and evaluate the integral for the flux across a smooth plane curve.

5. Find the work done by the force **F**(x, y, z) = (y + z)**i** + (z + x)**j** + (x + y)**k** along the path $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}, 0 \leq t \leq 1$.

6. Find the circulation and the flux for the field **F**(x, y) = -y**i** + x**j**, around the closed path consisting of an arch r_1(t) $= 3cos(t)\mathbf{i} + 3sin(t)\mathbf{j}, 0 \leq t \leq \pi$ followed by the line segment connecting (-3, 0) and (3, 0) (which defines r_2(t)).