Sample Exam 3 for 16C Hillel Raz

Note that this is a sample exam and does not reflect the length of the actual test. The actual midterm will be shorter, though the types of questions will be similar and based on the material covered here. The questions on the exam might be harder or easier, the difficulty here is not necessarily representative. It would probably be wise to do more practice of these problems, do extra problems from the book - the sections of the chapter review!

Series and Taylor Polynomials (Chapter 10.1-10.4)

- Know how to find the nth term of a sequence (via the pattern), find the limit of the sequence and hence determine the convergence or divergence of the sequence.

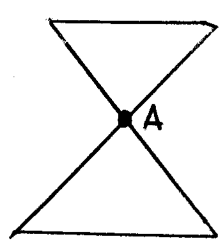
1. Find the pattern of the following sequences, their limit and discuss their convergence:

a) 1, -3, 7, -15, 31, ...

b) $1, \frac{3}{5}, \frac{3}{7}, \frac{1}{3}, \ldots$

c) $3, \frac{6}{2}, \frac{11}{6}, \frac{18}{24}, \frac{27}{120}, \ldots$

Challenge version of a sequence question: What is the maximum number of triangles that can be drawn in the figure below using 200 lines going through the point A (The meeting point of the two traingles)? Make sure to count the overlapping triangles as well. (Hint: try to write out the number of triangles as a sequence for one line (n=1), two lines, (n=2),...).



- Know the difference between a series and a sequence.
- Know the definition of the nth term test and how to use it.
- Know how to find the partial sum of a sequence and how to use it to figure out whether the sequence converges or diverges.
- Be very familiar with the geometric series: the partial sums of it, convergence of it, how to use it in word problems.
- Know the properties of series and how to use them in a time of need...

2. (once again, only one question 2 this time...) For the following series, decide whether they converge or diverge. Explain your reasoning. If the series converges, find when possible, what it converges to.

a) $\sum_{n=1}^{\infty} \frac{n^2+1}{n(n+1)}$

b) $\sum_{i=-1}^{20} i$

c) $\sum_{n=1}^{\infty} \frac{15n^3 - 2n^6 + 10001}{n^6 - 42n^2 + \pi n^2 - 4}$

d) $\sum_{k=0}^{\infty} 6(-\frac{4}{3})^k$

e) $\sum_{n=0}^{\infty} [(0.5)^n + (0.2)^n]$

f) $\sum_{n=5}^{72} 4(-\frac{1}{4})^n$

g) $0.18 + 0.0018 + 0.000018 + 0.00000018 + ...$

h) Number 41, p. 713, or similar problems (not necessarily a bouncing ball though...).

For more practice look in the book, both in section 10.2 or in the chapter review!

- Know how to recognize a p-series, and when it converges.
- Know the Harmonic Series and be able to explain why it diverges.
- Know the definition of the ratio test, how to use it and be able to tell using it whether a series converges or diverges.

3. For the following series, decide whether they converge or diverge. Explain your reasoning. If the series converges, find when possible, what it converges to.

a) $\sum_{n=1}^{\infty} \frac{1}{2n}$

b) $\sum_{n=5}^{\infty} \frac{n4^n}{n!}$

c) $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt[3]{n^{10}}}$

d) $\sum_{n=3}^{\infty} \frac{2n}{1 - 4^n}$

e) $\sum_{n=1}^{\infty} (5n! + \frac{10}{(n+2)!})$

For more of these to practice, look in the book section 10.3 and chapter review! Highly recommended... Also, note that on the exam, all of the questions asking about series convergence will be grouped together, rather than according to sections.

- Be familiar with the definition of a power series.
- Know how to calculate the radius of convergence of a power series.

4. For the following power series, discuss when the series converges/diverges and find the radius of convergence.

a) $\sum_{n=1}^{\infty}(2x)^n$

b) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(x+1)^n}{4^n}$

c) $\sum_{n=1}^{\infty}\frac{3x^n}{(n+5)(n+10)}$

d) $\sum_{n=0}^{\infty}\frac{(x+1)^n}{(n+1)!}$

- Know how to find a Taylor series of a function.
- Know how to use the shortcuts, including integration and differentiation, to get from one Taylor series to a related one.

5. For the following functions, find their Taylor series. Memorizing a Taylor series for a function and not showing any work of how one would get it will not get you any points on the exam...

a) Find the Taylor series for e^x centered at 0.

b) Use your answer from part a) to find the Taylor series for e^{-x^3}.

c) Find the Taylor series for $\frac{1}{x+1}$ centered at 0.

d) Use your answer from c) to find the Taylor series for $\frac{1}{(x+1)^3}$.

e) Find the Taylor series for $sinx$ and for $cosx$.