Ex. Analyze the graph of $f(x) = \dfrac{x^2+1}{x^2-1}$ and sketch it.

1) Domain: note that $\dfrac{x^2+1}{x^2-1} = \dfrac{x^2+1}{(x+1)(x-1)}$

$\Rightarrow$ discont. at $x = 1,\ x = -1$. (asymptotes!)

$x \to 1^-,\ f(x) \to -\infty \qquad x \to 1^+,\ f(x) \to +\infty$

$x \to -1^-,\ f(x) \to +\infty, \qquad x \to -1^+,\ f(x) \to -\infty.$

~~asymptotes~~ x-int? $\quad y = 0 \overset{?}{=} \dfrac{x^2+1}{(x+1)(x-1)} \Rightarrow x^2+1 \overset{?}{=} 0,$

so $x^2 \overset{?}{=} -1$ – never – no x-int.

2) max, min (Critical points).

$$f'(x) = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2} \quad \text{(simplify).}$$

$f'(x) = 0$ when $x = 0$.

$f'(x)$ undefined when $x = \pm 1$.

3) pts. of inflection

$$f''(x) = \frac{-4((x^2-1)^2) + 4x(2(x^2-1)2x}{(x^2-1)^4}$$

(pulled out $-4(x^2-1)$ – change signs!)

$$= \frac{-4(x^2-1)(x^2-1-(4x^2))}{(x^2-1)^4} = \frac{-4\cancel{(x^2-1)}(-3x^2-1)}{(x^2-1)^{\cancel{4}\,3}}$$

$$= \frac{-4(-3x^2-1)}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3}$$

$f''(x)$ never equals 0, undefined at $x = \pm 1$ – which are the asymptotes.

4) Intervals of ↑, ↓, concavity:

	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
$x =$	-2	$-\frac{1}{2}$	$\frac{1}{2}$	$x=2$
$f'(x)$	>0 ↑	>0 ↑	<0 ↓	<0 ↓
$f''(x)$	>0 concave up	<0 concave down	<0 concave down	>0 concave up

$x=0$ max.

$f(0) = -1$.

Sketch