Errata

Applied Analysis

(Corrected in Second Printing)

- p. 9: line 2 from the bottom: $\sqrt{2}$ instead of 2.
- p. 10: Last sentence should read: "The $\limsup$ of a sequence whose terms are bounded from above is finite or $-\infty$, and the $\liminf$ of a sequence whose terms are bounded from below is finite or ∞."
- p. 12, line 5 from the bottom: "This property is, in general, not equivalent to ...".
- p. 39 Example 2.7 Replace $C_0(\mathbb{R})$ by $C_0(\mathbb{R}^n)$, so it reads "The space $C_0(\mathbb{R}^n)$ consists of...there is an $R > 0$ such that $\|x\| > R$ implies that..."
- p. 46: Delete all of the paragraph above Example 2.13 except for the first sentence i.e delete from "The same proof applies to a more general situation..." to "...if and only if it is closed, bounded, and equicontinuous."
- p. 46: Add the following sentence at the bottom of the page, below (2.13): "These functions consist of 'tent' functions of height one that move from right to left across the interval $[0,1]$, becoming narrower and steeper as they do so."
- p. 47, line 4 from the bottom: replace "intermediate" by "mean".
- p. 59, Exercise 2.7, 2nd line: "Lipschitz constant less than or equal to one ..."
- p. 60, Exercise 2.10, line 3, "... the space of continuous functions that ..."
- p. 76 Hence if we choose $\eta = 1/(2C)$...
- p. 78 Exercise 3.1:

$$|T(x) - T(y)| < |x - y| \qquad \text{for all } x \neq y \in \mathbb{R}$$

- p. 79 Exercise 3.5: $\|L + U\|_\infty < \|D\|_\infty$

- p. 83, 1st line after Theorem 4.7: replace "defined" by "characterized".
- p. 89, Exercise 4.5, add "non-empty": ".. of two non-empty open sets."
- p. 89, Exercise 4.6: remove the last sentence, "Note that..."
- p. 94, line 3 & 4 from the bottom: the intervals I_n and J_n should be:

$$\begin{aligned} I_n &= [2^{-k}(2n-2), 2^{-k}(2n-1)), \\ J_n &= [2^{-k}(2n-1), 2^{-k}2n) \end{aligned}$$

- p. 100, line 13: line should end as follows: "...all $x \in M$. Moreover, $\|\overline{T}\| = \|T\|$."
- p. 109, last line of Definition 5.39: "... in the uniform, or operator norm, topology ...".
- p. 110 Proposition 5.43 should be rewritten as follows:

  Proposition 5.43 Let X, Y, Z be Banach spaces. (a) If $S, T \in \mathcal{B}(X, Y)$ are compact, then any linear combination of S and T is compact. (b) If (T_n) is a sequence of compact operators in $\mathcal{B}(X, Y)$ converging uniformly to T, then T is compact. (c) If $T \in \mathcal{B}(X, Y)$ has finite-dimensional range, then T is compact. (d) Let $S \in \mathcal{B}(X, Y)$, $T \in \mathcal{B}(Y, Z)$. If S is bounded and T is compact, or S is compact and T is bounded, then $TS \in \mathcal{B}(X, Z)$ is compact.

- p. 110, line 4 from bottom: replace "(a)–(c)" by "(a)–(b)"
- p. 111, 1st line: replace "(c)–(d)" by "(b)–(c)"
- p. 111, 2nd line after Definition 5.44: replace " norm on X", by " norm on Y".
- p. 116, in equation (5.24), change dummy index to j:

$$\omega_i \left(\sum_{j=1}^{n} x_j e_j \right) = x_i$$

- p. 116, 3rd line form the bottom, insert commas: "dual space, $\varphi : X \to \mathbb{R}$, ..."

- p. 119 ...and we say that X is *reflexive.*
- p. 119, line 5, " ... space."
- p. 121, Exercise 5.6, part (a) should start: "For any non-zero $x \in X$, ..."
- p. 132, last displayed equation should be

$$\langle z, y - y' \rangle = \langle z, x - y' \rangle - \langle z, x - y \rangle = 0.$$

- p. 136, line 3 of the 2nd paragraph, replace "One can show" by "It is easy to see".
- p. 136, first sentence of 3rd paragraph should read: "... converges to x, then for each $n \in \mathbb{N}$, there is a finite $J_n \subset I$ such that for all J containing J_n, one has $\|S_J - x\| \leq 1/n$."
- p. 140, line 3 should start: " To show that every Hilbert space has an orthonormal basis, we use ..."
- p. 140, 2nd paragraph after the "proof", 2nd line, replace "orthogonalization" by "orthonormalization".
- p. 145 Exercise 6.11: Prove that if $\mathcal{M}$ is a dense linear subspace of a separable Hilbert space $\mathcal{H}$...
- p. 184, Exercise 7.7, line 4, "$0 \leq x \leq 1$" should be "$0 \leq x \leq L$".
- p. 186, line 2, displayed equation should be:

$$x_{n+1} = \alpha x_n \pmod 1,$$

- p. 191, line 3 of the proof : "... $P : \mathcal{H} \to \mathcal{H}$ by"
- p. 195, line 9, "...if A is a bounded...".
- p. 195, in Theorem 8.18, "Suppose".
- p. 199 Sentence starting line 6 from bottom should read: "A map $U : \mathcal{H} \to \mathcal{H}$ is unitary if and only if $U^*U = UU^* = I$." Delete the rest of the sentence from "...meaning that..."

- p. 200, line 6, remove the second occurrence of "bounded", to read "bounded, skew-adjoint operators"

- p. 203, line 17, "... unitary. The subspace of functions invariant under U consists ..."

- p. 205, line 12, Replace "$x_1 \in B(0,1)$" by "$x_1 \in B(x_0, r)$".

- p. 205, last line: "linear functionals".

- p. 207 Replace first sentence of the last paragraph by: "As the above examples show, the norm of the limit of a weakly convergent sequence may be strictly less than the norms of the terms in the sequence, corresponding to a loss of "energy" in oscillations, at a singularity, or by escape to infinity in the weak limit."

- p. 212 Exercise 8.1. Change last sentence in Part (c) to: "Is a subspace of a Banach space with finite codimension necessarily closed?"

- p. 212, Replace Exercise 8.4 by the following problem. "Suppose that (P_n) is a sequence of orthogonal projections on a Hilbert space $\mathcal{H}$ such that

  $$\operatorname{ran} P_{n+1} \supset \operatorname{ran} P_n, \qquad \bigcup_{n=1}^{\infty} \operatorname{ran} P_n = \mathcal{H}.$$

  Prove that (P_n) converges strongly to the identity operator I as $n \to \infty$. Show that (P_n) does not converge to the identity operator with respect to the operator norm unless $P_n = I$ for all sufficiently large n."

- p. 214, Exercise 8.19 should read: "Prove that a strongly lower-semicontinuous convex function $f : \mathcal{H} \to \mathbb{R}$ on a Hilbert space $\mathcal{H}$ is weakly lower-semicontinuous."

- p. 233, line 15, add $k \geq 1$ in the displayed formula:

  $$\text{...for } n \neq m,\ k \geq 1.$$

- p. 240, line 1, insert "complex": " ... on a complex Hilbert space."

- p. 240, Exercise 9.7: add item: (d) Show that 0 belongs to the continuous spectrum of K.

- p. 241 Beginning of Exercise 9.13 should read: "Suppose that $L : \mathbb{R} \to \mathcal{B}(\mathcal{H})$ and $A : \mathbb{R} \to \mathcal{B}(\mathcal{H})$..."

- p. 242, Exercise 9.18 should begin: "Suppose that A is a compact self-adjoint operator. Let $f \in C(\sigma(A))$, and ..."

- p. 243, Exercise 9.19 should begin: "Let A be a compact selfadjoint linear operator. Prove ...".

- p. 246, 3rd line from the bottom: "theorm" should be "theorem"

- p. 253, in equation (10.11), add dy:

$$Gf(x) = \int_0^1 g(x,y)f(y)dy$$

- p. 256, line 18, "we choose nonzero solutions v_1 and v_2..."

- p.274 Theorem 10.35: $u\Delta v - v\Delta u = \nabla \cdot (u\nabla v - v\nabla u)$

- p. 280

$$\begin{aligned}\int_0^T \int_\Omega (-u_t + \Delta u)\, v \, dx dt &= \int_0^T \int_\Omega u\,(v_t + \Delta v)\; dx dt \\ &\quad - \int_\Omega [uv]_0^T \, dx + \int_0^T \int_{\partial\Omega} \left(v\frac{\partial u}{\partial n} - u\frac{\partial v}{\partial n} \right) dS dt.\end{aligned}$$

- p. 284, Exercise 10.15: in the first displayed equation "$L^1(\mathbb{R})$" should be replaced by "$L^1(\mathbb{R}) \cap L^2(\mathbb{R})$", and the sentence after that equation should read: "Show that A is a densely defined unbounded linear operator in $L^2(\mathbb{R})$ that is *not* closed."

- p. 284, 3rd line from the bottom: "velcity" should be "velocity".

- p. 300

$$\int_{|x|\geq 1} \frac{\sin nx}{\pi x}\phi(x)\, dx = \frac{1}{n}\left[\cos nx \frac{\phi(x)}{x}\right]_{-1}^{1} + \frac{1}{n}\int_{|x|\geq 1} \cos nx \left(\frac{\phi(x)}{x}\right)' \, dx.$$

- p. 309, line 2 from the bottom, at end of the line dx should be replaced by dy, so end of formula reads $g(y)\, dy$.

- p. 318
$$g(x,t) = \frac{1}{(2\pi t)^{n/2}} e^{-|x|^2/(2t)}$$

- p. 320 This equation may be interpreted as the Fourier series expansion...

- p. 330, Exercise 11.13, 2nd sentence should read: "Prove the corresponding results for derivatives and translates of tempered distributions and for the convolution of a test function with a tempered distribution."

- p. 331, Exercise 11.19.Replace last f by $\hat{f}$, to read "That is, find a function $f \in L^2(\mathbb{R})$ such that $\hat{f}$ is not continuous."

- p. 361, Theorem 12.59. The first line should start : "If $1 < p < \infty$, then ..." On the 3rd line of the theorem, before "Moreover", insert the sentence "If μ is σ-finite the same conclusion holds when $p = 1$ and $p' = \infty$."

- p. 361 In first line of Example 12.61, replace "$1 \leq p < \infty$" by "$1 < p < \infty$".

- p. 362 In line 2 of Theorem 12.62, replace "$1 \leq p < \infty$" by "$1 < p < \infty$". Delete the last sentence of the Theorem starting "If $p = \infty$..."

- p. 364 The action of this distribution f on a $W_0^{k,p'}$-function u...

- p. 373 Last sentence in Theorem 12.81 should read:

  Then for every bounded linear functional $F : \mathcal{H} \to \mathbb{C}$ there is a unique element $x \in \mathcal{H}$ such that
$$a(x,y) = F(y) \qquad \text{for all } y \in \mathcal{H}.$$