HW 1 Solutions
Math 118: PDE

1.1.2 Linear: (a), (e). Nonlinear: (b), (c), (d).
1.1.3 (a) second; linear inhomogeneous. (b) second; linear homogenous. (c) third; nonlinear. (d) second; linear inhomogenous. (e) second; linear homogenous. (f) first; nonlinear. (g) first; linear homogenous. (h) fourth; nonlinear.
1.1.10 Recall that a vector space is closed under addition and scalar multiplication. Take two soultions u, v of the DE. Show that $u+v$ and cu where c is a scalar constant are also solutions. Characteristic equation of the DE is $r^3 - 3r^2 + 4 = (r+1)(r-2)^2 = 0$. A basis of the DE is thus $\{e^{-1}, e^2, te^2\}$.
1.1.12 Recall that

$$\frac{d}{dx}\sinh x = \cosh x, \quad \frac{d}{dx}\cosh x = \sinh x,$$

where

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}.$$

1.2.1 Chatacteristic lines are $3t - 2x = C$, where C is an arbitrary constant. Thus, the solution has the form

$$u(t, x) = f(3t - 2x).$$

When $t = 0$, we have

$$u(0, x) = f(-2x) = \sin x \implies f(x) = \sin(-x/2).$$

That is, the solution is

$$u(t, x) = \sin(x - 3t/2).$$

1.2.4 Direct substitution.
1.2.9 Change variables to $x' = x + y, y' = x - y$, follow examples on Page 7, we obtain

$$2u_{x'} = 1 \implies u = \frac{1}{2}x' + f(y') = \frac{1}{2}(x+y) + f(x-y).$$