Midterm 1: Sample questions
Math 118A, Fall 2013

1. Say whether the following operators acting on functions $u(x, y)$ are linear or nonlinear. Justify your answers. (a) $Lu = u_{xx} + u_{yy} + 1$; (b) $Lu = yu_{xx} + u_{yy} + u$; (c) $Lu = uu_{xx} + u_{yy}$.

2. Solve the following IVP for the wave equation

$$u_{tt} = c^2 u_{xx}, \qquad u(x, 0) = 0, \quad u_t(x, 0) = \cos x.$$

3. Look for solutions of the heat equation

$$u_t = ku_{xx},$$

of the form

$$u(x, t) = f(x)e^{-a^2 t}$$

where $a > 0$ is a constant. Find the most general function $f(x)$ for which this is a solution. Give a physical explanation, in terms of heat flow, of why this solution decays exponentially in time.

4. For what values of the constants m, n does the PDE

$$u_t + uu_x + u_{xxx} = 0$$

have similarity solutions of the form

$$u(x, t) = \frac{1}{t^m} f\left(\frac{x}{t^n}\right)?$$

In that case, find an ODE for $f(z)$. (Don't try to solve it!)

5. Suppose that $u(x, t)$ is a solution of the initial value problem

$$u_t + cu_x + u = 0, \qquad u(x, 0) = \phi(x).$$

such that u and its derivatives approach zero as $|x| \to \infty$. Show that

$$\int_{-\infty}^{\infty} u^2(x, t)\, dx = e^{-2t} \int_{-\infty}^{\infty} \phi^2(x)\, dx.$$

6. Suppose that algae on a (one-dimensional) lake has population density $u(x, t)$. Assume that the algae grows at a rate proportional to its population density and diffuses from high-density to low density regions at a rate proportional to its population gradient u_x. Derive a PDE for $u(x, t)$.