Problem Set 2: Math 118B

Winter Quarter, 2014

1. (a) Find the Green's function $G(x)$ for the ODE

$$-\frac{d^2G}{dx^2} + G = \delta(x) \qquad -\infty < x < \infty$$

where $G(x) \to 0$ as $|x| \to \infty$.

(b) Write down the Green's function representation of the solution $u(x)$ of

$$-\frac{d^2u}{dx^2} + u(x) = f(x) \qquad -\infty < x < \infty$$

where $u(x) \to 0$ as $|x| \to \infty$ and $f(x)$ is a given (smooth) function that is zero outside a bounded set.

(c) Verify explicitly that the your expression for $u(x)$ in (b) is a solution.

(d) Give a physical interpretation of this problem (in terms of heat flow, for example).

2. (a) Find the Green's function $G(x;\xi)$ for the BVP

$$-\frac{d^2G}{dx^2} = \delta(x-\xi), \qquad 0 < x < 1$$
$$G(0;\xi) = 0, \qquad G(1;\xi) = 0,$$

where $0 < \xi < 1$. (Note: In this problem $G(x;\xi)$ isn't a function of $x - \xi$ because of the boundary conditions.)

(b) Sketch the graph of the Green's function $G(x;\xi)$ versus x for a few different values of ξ. Give a physical interpretation of the BVP in terms of: (i) heat flow; (ii) an elastic string. Does the Green's function look the way you would expect?

(c) Use the superposition principle to explain why you expect the solution of the BVP

$$-\frac{d^2u}{dx^2} = f(x), \qquad 0 < x < 1$$
$$u(0) = 0, \qquad u(1) = 0$$

to have the Green's function representation

$$u(x) = \int_0^1 G(x;\xi) f(\xi)\, d\xi.$$

(d) Evaluate the Green's function representation for $u(x)$ in (c) explicitly if $f(x) = \sin \pi x$. Verify that it gives the solution of the BVP.