Sample Midterm Questions
Math 121, Fall 2004

1. Use Fourier series to find the solution $u(x, y)$ of the following boundary value problem for Laplace's equation in the semi-infinite strip $0 < x < 1$, $y > 0$:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \\
&u(0, y) = u(1, y) = 0, \\
&u(x, 0) = 1, \\
&u(x, y) \to 0 \quad \text{as } y \to \infty.
\end{aligned}$$

2. Use Fourier series to find the solution $u(x, t)$ of the following initial-boundary value problem for the wave equation in $0 < x < 1$ and $t > 0$:

$$\begin{aligned}
&\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \\
&\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(1, t) = 0, \\
&u(x, 0) = 0, \\
&\frac{\partial u}{\partial t}(x, 0) = x.
\end{aligned}$$

3. Use Fourier transforms to solve the following initial value problem for $u(x, t)$ in $-\infty < x < \infty$, $t > 0$:

$$\begin{aligned}
&\frac{\partial u}{\partial t} = -\frac{\partial^4 u}{\partial x^4}, \\
&u(x, 0) = f(x).
\end{aligned}$$

Write the solution for $u(x, t)$ as a convolution, but do compute any inverse transforms explicitly. How smooth is the solution for $t > 0$?

4. (a) Give the formulas for the Fourier transform $\widehat{f}(k)$ of a function $f(x)$ and the inverse Fourier transform.

(b) Compute the Fourier transform of $e^{-|x|}$.

(c) State Parseval's theorem, and use it to evaluate

$$\int_0^\infty \frac{1}{(1+k^2)^2}\,dk.$$

5. Use Laplace transforms to solve the following initial value problem:

$$\begin{aligned} &y'' + 2y' + 2y = 1, \\ &y(t) = 0, \quad y'(0) = 1. \end{aligned}$$

6. (a) Say what jump conditions the solution of $y(t)$ of the following initial value problem satisfies at $t = 0$, and find the solution directly (do not use Laplace transforms):

$$\begin{aligned} &y'' - 4y = \delta(t), \\ &y(t) = 0 \quad \text{for } t < 0. \end{aligned}$$

(b) Write the solution of the following initial value problem, where $f(t)$ is an arbitrary function, as a convolution (you don't need to derive your answer):

$$\begin{aligned} &y'' - 4y = f(t), \\ &y(0) = y'(0) = 0. \end{aligned}$$