

An Introduction to Real Analysis

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ABSTRACT. These are some notes on introductory real analysis. They cover limits of functions, continuity, differentiability, and sequences and series of functions, but not Riemann integration. A background in sequences and series of real numbers and some elementary point set topology of the real numbers is assumed, although some of this material is briefly reviewed.

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