Sample Questions for Midterm 1
Math 125A, Fall 2012

Closed Book. Give complete proofs of all your answers. Unless stated otherwise, you can use any standard theorem provided you state it clearly.

1. (a) Suppose that $f : (0,1) \to \mathbb{R}$ is uniformly continuous on $(0,1)$. If (x_n) is a Cauchy sequence in $(0,1)$ and $y_n = f(x_n)$, prove that (y_n) is a Cauchy sequence in $\mathbb{R}$.

(b) Give a counter-example to show that the result in (a) need not be true if $f : (0,1) \to \mathbb{R}$ is only assumed to be continuous.

2. (a) State the ϵ-δ definition for a function $f : \mathbb{R} \to \mathbb{R}$ to be continuous at $c \in \mathbb{R}$.

(b) Define the floor function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \text{the largest integer } n \in \mathbb{Z} \text{ such that } n \leq x.$$

For example, $f(3.14) = 3$, $f(7) = 7$, $f(-3.14) = -4$. Determine, with proof, where f is continuous and where it is discontinuous.

3. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a continuous function such that

$$\lim_{x \to -\infty} f(x) = 0, \qquad \lim_{x \to \infty} f(x) = 0.$$

(a) Give a precise statement of what these limits mean.

(b) Prove that f is bounded on $\mathbb{R}$ and attains either a maximum or minimum value.

(c) Give examples to show that f may: (i) attain its maximum but not its infimum; (ii) attain both its maximum and minimum.