Midterm 2: Sample questions
Math 125B: Winter 2013

1. Suppose that $f : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by

$$f(x, y, z) = \left(x^2 + yz, \sin(xyz) + z\right).$$

(a) Why is f differentiable on $\mathbb{R}^3$? Compute the Jacobian matrix of f at $(x, y, z) = (-1, 0, 1)$.
(b) Are there any directions in which the directional derivative of f at $(-1, 0, 1)$ is zero? If so, find them.

2. Suppose that $f : \mathbb{R} \to \mathbb{R}^3$ and $g : \mathbb{R}^3 \to \mathbb{R}$ are defined by

$$f(t) = (t, t^2, t^3), \qquad g(x, y, z) = x^2 e^{yz},$$

and $h = g \circ f : \mathbb{R} \to \mathbb{R}$ is their composition.
(a) Use the chain rule to compute $h'(1)$.
(b) Find $h(t)$ and compute $h'(1)$ directly.

3. Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = \begin{cases} x^{4/3} \sin(y/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Where is f is differentiable?

4. Suppose that $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $x \in \mathbb{R}^n$. If $A : \mathbb{R}^m \to \mathbb{R}^p$ is a linear map, prove from the definition of the derivative that $Af : \mathbb{R}^n \to \mathbb{R}^p$ is differentiable at x and find its derivative. (You can assume that $\|Ah\| \leq M\|h\|$ for some constant M. See p. 212 of the text)

5. The mean value theorem from one-variable calculus states that if a function $f : [a, b] \to \mathbb{R}$ is continuous on the closed interval $[a, b]$ and differentiable in the open interval (a, b), then there is a point $a < c < b$ such that

$$f(b) - f(a) = (b - a) f'(c).$$

Does this theorem remain true for a vector-valued function $f : [a, b] \to \mathbb{R}^2$?