Problem Set 4: Additional questions
Math 125B: Spring 2013

1. Suppose that $f_n, f : [a, b] \to \mathbb{R}$ are integrable functions and $f_n \to f$ as $n \to \infty$ uniformly on $[a, b]$. Let

$$F_n(x) = \int_a^x f_n(t)\, dt, \qquad F(x) = \int_a^x f(t)\, dt.$$

Prove that $F_n \to F$ uniformly on $[a, b]$. If $f_n \to f$ pointwise, does it follow that $F_n \to F$ pointwise?

2. Let $f_n : (a, b) \to \mathbb{R}$ be a sequence of differentiable functions whose derivatives $f_n' : (a, b) \to \mathbb{R}$ are integrable on (a, b). Suppose that $f_n \to f$ pointwise and $f_n' \to g$ uniformly on (a, b) as $n \to \infty$, where $f, g : (a, b) \to \mathbb{R}$ and g is continuous. Prove that f is differentiable in (a, b) with $f' = g$. Where do you need to use the continuity of g?

Hint. Show that the indefinite integrals of f_n' converge and use the fundamental theorem of calculus.