Sample Questions
Midterm II
Math 127B. Winter, 2005

Closed Book. No calculators.
Except in Question 1, give complete proofs of all your answers.
You can use any standard theorem provided you state it carefully.

1. For each of the following statements, say if it is true or false. (No explanation is required.)

(a) If f is differentiable and $f' > 0$, then f is strictly increasing.

(b) If f is strictly increasing and differentiable, then $f' > 0$.

(c) If f is the sum of a convergent Taylor series in an open interval containing the origin, then f is infinitely differentiable.

(d) If f is infinitely differentiable in an open interval containing the origin, then the Taylor series of f converges.

(e) There exists $0 < x < 1$ such that $e^x \sin 1 = \cos x\,(e-1)$.

2. Define the derivative. Consider

$$f(x) = \begin{cases} x^a & \text{for } x \text{ irrational,} \\ 0 & \text{for } x \text{ rational.} \end{cases}$$

For what values of $a > 0$ is f differentiable at 0? Is f differentiable at $x \neq 0$?

3. State Taylor's theorem. Prove that

$$\log(1+x) < x$$

for all $x > 0$.

4. Carefully state a version of L'Hospital's rule that applies to the following limit. Use it to prove that the limit exists, and find its value:

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}.$$

5. Define the hyperbolic sine

$$\sinh x = \frac{e^x - e^{-x}}{2}.$$

Prove that $\sinh x$ is strictly increasing on $\mathbb{R}$ and hence has an inverse. Prove that the inverse is differentiable and compute its derivative.

6. A function f has a jump discontinuity at x_0 if both the left and right limits

$$\lim_{x \to x_0^+} f(x), \qquad \lim_{x \to x_0^-} f(x)$$

exist but have different values. Suppose that $f : (a, b) \to \mathbb{R}$ is differentiable in (a, b). Prove that f' does not have a jump discontinuity in (a, b).