

Sample Midterm Questions 2
Math 127C, Spring 2006

1. Show that

$$d(x, y) = \frac{|x - y|}{1 + |x - y|}$$

is a metric on \mathbb{R} . Is \mathbb{R} complete with respect to this metric?

2. Does the equation

$$x^5 + y^5 + xy + 4 = 0$$

define an implicit function $x = g(y)$ locally near the point $(x, y) = (-2, 2)$? Explain your answer.

3. Suppose that $1/2 \leq a \leq 3/2$. Define a function $\phi : \mathbb{R} \rightarrow \mathbb{R}$ by

$$\phi(x) = x + \frac{1}{2}(a - x^2)$$

Find a closed bounded interval $I \subset \mathbb{R}$ containing 1 such that $\phi : I \rightarrow I$ is a contraction. If $x_0 \in I$, what do the iterates

$$x_{n+1} = x_n + \frac{1}{2}(a - x_n^2)$$

converge to as $n \rightarrow \infty$?

4. Use the change of variables formula to transform

$$\int_0^\infty \int_0^\infty e^{-x^2 - y^2} dx dy$$

into an integral with respect to polar coordinates (r, θ) , where

$$x = r \cos \theta, \quad y = r \sin \theta.$$

Deduce the value of

$$\int_0^\infty e^{-x^2} dx$$

Justify your steps.