

Problem Set 7

Math 205A: Winter Quarter, 2014

1. Prove the following addition theorem for the Weierstrass \wp -function:

$$\wp(a+b) + \wp(a) + \wp(b) = \frac{1}{4} \left[\frac{\wp'(a) - \wp'(b)}{\wp(a) - \wp(b)} \right]^2.$$

2. Deduce that the (non-singular, projective) elliptic curve \mathcal{C} with equation

$$w^2 = 4z^3 - g_2z - g_3$$

is a group with respect to the following addition operation: if $A, B \in \mathcal{C}$, let $C' \in \mathcal{C}$ be the third intersection point of the line through A, B with \mathcal{C} (counting multiplicity), and let $C = A + B$ be the reflection of C' in the z -axis ($w \mapsto -w$). The identity element is the point at infinity, and a line through infinity is a vertical line $z = \text{constant}$.