Problem set 1
Math 207A, Fall 2011
Due: Wed., Oct. 5

1. Write the IVP for the forced, damped pendulum

$$x_{tt} + \delta x_t + \omega_0^2 \sin x = \gamma \cos \omega t,$$
$$x(0) = x_0, \qquad x_t(0) = v_0$$

as an IVP for an autonomous first-order system. What is the dimension of the system?

2. Solve the scalar IVP

$$x_t = x(\log x)^\alpha, \qquad x(0) = x_0$$

where $\alpha > 0$ and $x_0 > 1$. Find the maximal time-interval on which the solution exists. For what values of α does the solution exist for all times?

3. The position $x(t) \in \mathbb{R}$ of a particle of mass m moving in one space dimension in a potential $V(x)$ satisfies

$$m x_{tt} = -V'(x)$$

where the prime denotes a derivative with respect to x. Show that the total energy

$$\frac{1}{2} m x_t^2 + V(x) = \text{constant}$$

is conserved. What can you say about the time-interval of existence of solutions for: (a) the attractive potential $V(x) = x^4$; (b) the repulsive potential $V(x) = -x^4$?

4. Linearize the Lorenz equations

$$x_t = \sigma(y - x),$$
$$y_t = rx - y - xz,$$
$$z_t = xy - \beta z$$

about the equilibrium solution $(x, y, z) = (0, 0, 0)$. Show that this equilibrium is linearly stable if $r < 1$ and linearly unstable if $r > 1$.