PROBLEM SET 2 Math 207A, Fall 2011 Due: Wed., Oct. 12

**1.** Solve the IVP for the logistic equation

$$x_t = x(1-x), \qquad x(0) = x_0.$$

2. Consider bacterial growth in a closed flask with a fixed initial amount of nutrient, and suppose that the growth rate of the bacteria is proportional to the amount of available nutrient. If N(t) denotes the population of bacteria and C(t) denotes the available nutrient at time t, explain why the ODEs

$$N_t = \mu C N, \qquad C_t = -\alpha \mu C N$$

provide a reasonable model for suitable constants  $\alpha, \mu > 0$ . Solve the system subject to the initial conditions

$$N(0) = N_0, \qquad C(0) = C_0$$

where  $N_0, C_0 > 0$ . Express the limiting population of bacteria

$$N_{\infty} = \lim_{t \to \infty} N(t)$$

in terms of  $\alpha$ ,  $\mu$ ,  $N_0$ ,  $C_0$ . Does your answer make sense?

**3.** Let

$$f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0, \\ 0 & x = 0. \end{cases}$$

Find the equilibria of the ODE  $x_t = f(x)$  and determine their stability, and sketch the phase line.

4. Graph the bifurcation diagram for equilibrium solutions of the scalar ODE

$$x_t = \mu + x - x^3$$

versus  $\mu$  and determine their stability. (You don't have to give an explicit expression for the equilibria.) Find the values of  $(x, \mu)$  at which equilibrium bifurcations occur. What kind of bifurcations are they? Sketch the phase line of the system for different values of  $\mu$ , including the values at which bifurcations occur. Describe what would happen if the system is in equilibrium and  $\mu$  is increased very slowly from  $\mu = -1$  to  $\mu = 1$  and then decreased back to  $\mu = -1$ .