Problem set 4
Math 207A, Fall 2011
Due: Wed., Oct. 26

1. Newton's method for the iterative solution of the scalar equation $f(x) = 0$ is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

If $f(x) = x^2 - 2$, show that this equation becomes

$$x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}.$$

What are the fixed points of this system? Determine their stability. Compute x_4 numerically if $x_0 = 3$.

2. Find the fixed points of the system

$$x_{n+1} = -\frac{\mu}{2} \tan^{-1} x_n$$

and determine their stability. Show that a period-doubling bifurcation occurs at $\mu = 2$. Is the resulting period-two orbit stable or unstable?

3. Consider the discrete dynamical system on the circle for $x_n \in \mathbb{T}$

$$x_{n+1} = x_n + \mu \qquad (\mathrm{mod}\ 2\pi)$$

corresponding to rotation by an angle $\mu \in \mathbb{T}$. Describe the structure of the orbits and how they depend on μ.

4. Carry out numerical experiments for iterations of the logistic map

$$x_{n+1} = \mu x_n (1 - x_n)$$

where $1 \leq \mu \leq 4$ and $0 \leq x_0 \leq 1$. (You can write your own program or use the MATLAB script provided on the course website.)