Problem set 7
Math 207A, Fall 2011
Due: Wed., Nov. 23

1. Classify the equilibrium $(x, y) = (0, 0)$ of the system

$$x_t = x, \qquad y_t = -y + x^2.$$

Is the equilibrium hyperbolic? Find an equation for the trajectories in (x, y)-phase space, and sketch the phase plane. What are the stable and unstable subspaces E^s and E^u and the stable and unstable manifolds $W^s(0, 0)$ and $W^u(0, 0)$ of the origin?

2. Write the system

$$x_t = x - y - x\left(x^2 + y^2\right)$$
$$y_t = x + y - y\left(x^2 + y^2\right)$$

in polar coordinates. Classify the equilibrium $(x, y) = (0, 0)$ and sketch the phase portrait. How do solutions behave as $t \to \infty$?

2. Find and classify the equilibria of the system

$$x_t = \mu x - x^2, \qquad y_t = -y.$$

Sketch the phase portraits for $\mu < 0$, $\mu = 0$, and $\mu > 0$. In each case, say if the equilibria are hyperbolic and describe their stable and unstable subspaces E^s and E^u and their stable and unstable manifolds W^s and W^u.

4. Consider the following model for the dynamics of a predator with population $x(t)$ and a prey with population $y(t)$ *e.g.* pikes and eels, or foxes and rabbits:

$$x_t = x\left(-1 + y\right),$$
$$y_t = y\left(1 - x\right).$$

Explain why this is a reasonable qualitative model for a predator-prey system. Find the equilibria and classify them. Sketch the phase portrait. How do solutions behave?

Hint. To find the trajectories, solve the first order ODE for y as a function of x that is obtained from

$$\frac{dy}{dx} = \frac{y_t}{x_t}.$$