PROBLEM SET 8
Math 207A, Fall 2011
Due: Fri., Dec. 2

1. Sketch the phase plane of the system

$$x_t = x^2, \qquad y_t = -y.$$

Linearize the system about the equilibrium $(0,0)$ and determine the unstable, stable and center subspaces of the equilibrium. What is the stable manifold $W^s(0,0)$? Show that there are many possible choices of a (C^1) center manifold $W^c(0,0)$.

2. Consider the Euler equations for a rotating rigid body

$$\begin{aligned}
\dot{M}_1 &= \left(\frac{1}{I_3} - \frac{1}{I_2}\right) M_2 M_3,\\
\dot{M}_2 &= \left(\frac{1}{I_1} - \frac{1}{I_3}\right) M_3 M_1,\\
\dot{M}_3 &= \left(\frac{1}{I_2} - \frac{1}{I_1}\right) M_1 M_2,
\end{aligned}$$

where $M_1(t)$, $M_2(t)$, $M_3(t)$ are components of the body angular momentum and the positive constants $0 < I_1 < I_2 < I_3$ are the moments of inertia of the body (which we assume to be distinct).

(a) Show that the (squared) total angular momentum

$$J = M_1^2 + M_2^2 + M_3^2$$

and the kinetic energy

$$T = \frac{M_1^2}{I_1} + \frac{M_2^2}{I_2} + \frac{M_2^2}{I_2}$$

are conserved.

(b) Restrict the Euler equations to the sphere

$$M_1^2 + M_2^2 + M_3^2 = 1,$$

which is an invariant manifold for the flow by (a). Find the equilibria on this sphere, linearize the equations about the equilibria, classify them, and determine their stability. Sketch the phase portrait on the sphere.