Problem set 4

Math 207A, Fall 2014

1. Sketch the bifurcation diagram and phase lines for the ODE

$$x_t = \mu x + x^3 - x^5,$$

and classify the bifurcations that occur. What would happen if the parameter μ is slowly increased from $-\infty$ to ∞, and then decreased back to $-\infty$? Where will the symmetry of the system under reflections $x \mapsto -x$ play a role?

2. Sketch the bifurcation diagrams and phase lines for x versus μ

$$x_t = a + \mu x - x^3$$

for: (i) $a < 0$; (ii) $a = 0$; (iii) $a > 0$. Classify the bifurcations that occur.

3. Sketch the bifurcation diagram and phase circles for the periodic ODE

$$x_t = -\mu + 2 + \cos 2x - 3\cos x,$$

and classify the bifurcations that occur. (See Exercise 2.14 in the text for more help.)

4. The following PDE for $u(x, t)$, called Burgers equation, is a simply model of the Navier-Stokes equations for viscous fluids

$$u_t + uu_x = u_{xx}$$

(a) Look for traveling wave solutions of the form $u = f(x - ct)$, and derive a first-order ODE for f.

(b) Show that the PDE has traveling wave solutions with $u \to u_-$ as $x \to -\infty$ and $u \to u_+$ as $x \to \infty$, where $u_\pm$ are constants, if $u_- \geq u_+$ and the wave-velocity c is given by

$$c = \frac{u_+ + u_-}{2}.$$

Solve for the traveling wave solution in that case.