PROBLEM SET 5
Math 207A, Fall 2014

0. Sketch "staircase pictures" in the (x, y) plane with graphs $y = ax$ and $y = x$ for one or two orbits of the linear scalar map

$$x_{n+1} = ax_n$$

in the following cases: (a) $a < -1$; (b) $a = -1$; (c) $-1 < a < 0$; (d) $a = 0$; (e) $0 < a < 1$; (f) $a = 1$; (g) $a > 1$.

1. Define $f : [0, 1] \to [0, 1]$ by

$$f(x) = \begin{cases} 3x & \text{if } 0 \le x \le 1/3 \\ \frac{3}{2}(1 - x) & \text{if } 1/3 < x \le 1 \end{cases}$$

and consider the discrete dynamical system

$$x_{n+1} = f(x_n).$$

(a) Find the fixed points of f in $[0, 1]$ and determine their stability.
(b) Find the period two orbit and determine its stability.

2. Find the fixed points of the system

$$x_{n+1} = -\frac{\mu}{2} \tan^{-1} x_n$$

and determine their stability. Show that a period-doubling bifurcation occurs at $\mu = 2$. Is the resulting period-two orbit stable or unstable?

3. Consider the discrete dynamical system on the circle for $x_n \in \mathbb{T}$

$$x_{n+1} = x_n + \mu \pmod{2\pi}$$

corresponding to rotation by an angle $\mu \in \mathbb{T}$. Describe the structure of the orbits and how they depend on μ.

4. Write a MATLAB script to solve the system

$$x_{n+1} = (1 - \mu)x_n + \mu x_n^3,$$

and investigate the dynamics of solutions. In particular, investigate numerically the sequence of period-doubling bifurcations that accumulates at $\mu \approx 3.5980\ldots$.